Parallel Variable Distribution for Mesh Adaptive Direct Search

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Presentation Outline

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Introduction

- MADS is a direct search algorithm for nonsmooth optimization
- PVD is a generic and parallel framework for optimization
- We propose to apply PVD to MADS in order to solve large problems ($n > 50$)

Target Problem

$$\min_{x \in \Omega \subseteq \mathbb{R}^n} f(x)$$

where

- $f : \mathbb{R}^n \to \mathbb{R} \cup \{\infty\}$
- function f and constraints defining Ω are black-box functions
  - nonsmooth
  - problematic derivative approximation
  - can possibly fail to evaluate
  - costly to evaluate
  - usually the result of a computer code

MADS Overview

- **M**esh **A**daptive **D**irect **S**earch [Audet, Dennis 2005]
- Extends the **G**eneralized **P**attern **S**earch [Torczon 1997]
- Direct Search method: derivative are not evaluated nor approximated
- Iterative algorithm where the black-box functions are evaluated at some trial points, which are either accepted as new iterates or rejected
- At iteration k: two steps: the Poll and the Search

MADS Mesh

- All trial points at iteration k are constructed to lie on a mesh

$$M(\Delta_k) = \left\{ x_k + \Delta_k Dz : z \in \mathbb{N}^{n_D} \right\} \subset \mathbb{R}^n$$

where $\Delta_k \in \mathbb{R}^+$ is the **mesh size parameter** and D a fixed set of n_D directions in $\mathbb{R}^n$

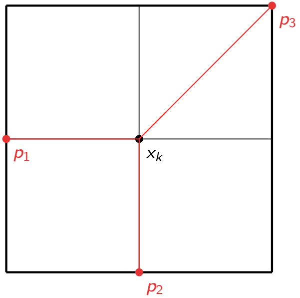
- After each iteration, Δ_k is reduced when no new iterate has been found (iteration fail)

Poll and Search

- Poll
  - local exploration on the mesh near the best current iterate x_k
  - use of MADS directions (at least one is necessary to ensure convergence)
- Search
  - global and flexible exploration strategy
  - has only to generate a finite number of trial points lying on the mesh

Poll illustration (successive fails and mesh shrink)

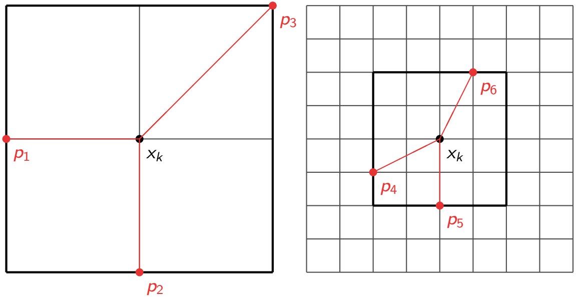
$\Delta_k = 1$



trial points=$\{p_1, p_2, p_3\}$

Poll illustration (successive fails and mesh shrink)

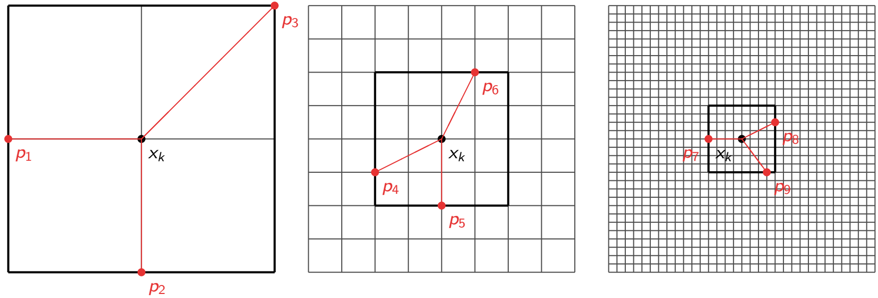
$\Delta_k = 1$ $\Delta_{k+1} = 1/4$



trial points$=\{p_1, p_2, p_3\}$ $= \{p_4, p_5, p_6\}$

Poll illustration (successive fails and mesh shrink)

$\Delta_k = 1$ $\Delta_{k+1} = 1/4$ $\Delta_{k+2} = 1/16$



trial points$=\{p_1, p_2, p_3\}$ $= \{p_4, p_5, p_6\}$ $= \{p_7, p_8, p_9\}$

MADS Convergence

- Constraints are handled with the barrier approach: if $x \notin \Omega$, $f(x)$ is considered to be $+\infty$
- A hierarchical convergence based on f differentiability analysis is available for MADS with barrier
- **Main convergence result** : MADS leads to a Clarke stationnary point $\hat{x} \in \Omega$ if f is Lipschitz near $\hat{x}$:
  $$f^{\circ}(\hat{x}; d) \geq 0 \text{ for all } d \in T_{\Omega}^{Cl}(\hat{x})$$
- **Corollary for unconstrained case** : if the function is strictly differentiable, then $\nabla f(\hat{x}) = 0$

PVD algorithm

- **P**arallel **V**ariable **D**istribution [Ferris, Mangasarian 1994]
- Generic and parallel optimization framework
- Parallelism achieved by a master/slaves paradigm
- Idea: each process works on a reduced problem and has responsability of small groups of variables
- Iterative algorithm with two steps:
  - **decomposition**: subproblems with a reduced number of variables are optimized in parallel
  - **synchronization**: results of subproblems are gathered; a new iterate is constructed

PVD algorithm for N processors

1 master, $(N-1)$ slaves

Initializations

x_0, lists of subproblems variables

Iteration k

[1] **Parallel Decomposition [by slave s_i]**

optimizes subproblem P_i from starting point x_k
$y_i \leftarrow$ solution of optimization

[2] **Synchronization [by master]**

$x_{k+1} \leftarrow$ new iterate from solutions y_i's
$k \leftarrow k+1$
goto [1] until a stopping condition is met

Adaptation of PVD for MADS

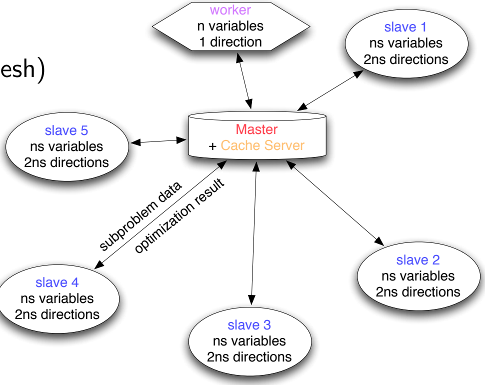
- MADS is used to optimize subproblems
- The synchronization barrier step is removed
- New algorithm most important parameters:
  - *bbe* : maximum number of black-box evaluations for each MADS optimization (does not include cache hits)
  - *ns* : number of variables in subproblems

Slaves and Worker

- Slaves: solve subproblems with standard MADS method, with a reduced number of variables
- Worker
  - is one of the slaves with special status
  - all n variables are considered
  - polls with only one MADS special direction: this direction is defined in the MADS method as the minimal direction allowing convergence

New method: master/slaves occupation

- Master
  - look for a slave signal
  - get optimization data
  - updates (current solution, mesh)
  - decide subproblem variables
  - send subproblem data
- Slave or Worker
  - receive subproblem data
  - optimize subproblem
  - send optimization data



Choice of sets L_i

- L_i are the sets of variables for subproblem P_i optimized by slave s_i
- Original PVD method:
  - fixed sets L_i for all iterations
  - sets L_i had to be a partition of $\{1, 2, ..., n\}$
- New method:
  - $L_i \longrightarrow L_i^k$ for variables of slave s_i at iteration k
  - sets have not to form a partition of $\{1, 2, ..., n\}$
  - sets are randomly and uniformly chosen
  - all sets have the same size ns

Mesh Update

- For all MADS optimizations, an initial mesh $M(\Delta_0)$ and a minimal mesh $M(\Delta_{min})$ are to be defined
- Initially: all slaves begin optimizations with user specified Δ_0
- Worker
  - $\Delta_0 \leftarrow$ last Δ_k of worker optimization
  - if the current solution has been updated by another slave, initial mesh is expanded (search success)
  - $\Delta_{min} \leftarrow$ user specified small value
  - $\Delta_{PVD} \leftarrow \Delta_0$
- Slaves
  - $\Delta_0 \leftarrow$ last Δ_k of slave optimization
  - $\Delta_{min} \leftarrow \Delta_{PVD}$: all trial points lie on mesh $M(\Delta_{PVD})$

Convergence

- **From the point of view of the Worker process**
- The Worker runs a complete MADS algorithm on the original problem :
  - A single direction Poll
  - Search
    - is performed after an overal *bbe* trial poll points
    - consists in obtaining the best iterate from slaves
    - by construction, slaves generate a finite number of points on the worker mesh
- All MADS convergence conditions are met: MADS theoretical convergence analysis holds

Test Problem [JOGO; Hedar, Fukushima]

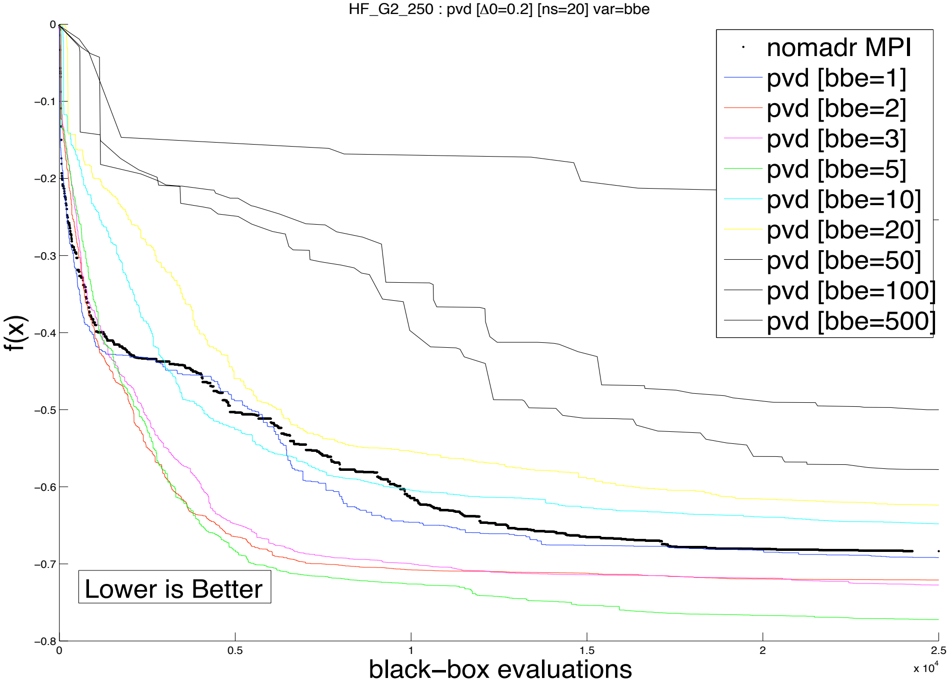
$$\min_{x \in \mathbb{R}^n} f(x) = \left| \frac{\sum_{i=1}^{n} \cos^4 x_i - 2 \prod_{i=1}^{n} \cos^2 x_i}{\sqrt{\sum_{i=1}^{n} i x_i^2}} \right|$$

$$s.t. \begin{cases} g_1(x) = -\prod_{i=1}^{n} x_i + 0.75 \leq 0 \\ g_2(x) = \sum_{i=1}^{n} x_i - 7.5n \leq 0 \end{cases}$$

$n = 250,\ 0 \leq x_i \leq 10,\ x_0 = [5\ 5\ \ldots\ 5]^T$

Testing protocols

- Graphs showing the number of evaluations *v.s* the objective function value
- Each plot is an average of 5 runs
- Different pvd runs are compared to a synchronous parallel MADS algorithm
- PVD parameters tested: *bbe* and *ns*
- Budget of 25000 evaluations
- 12 slaves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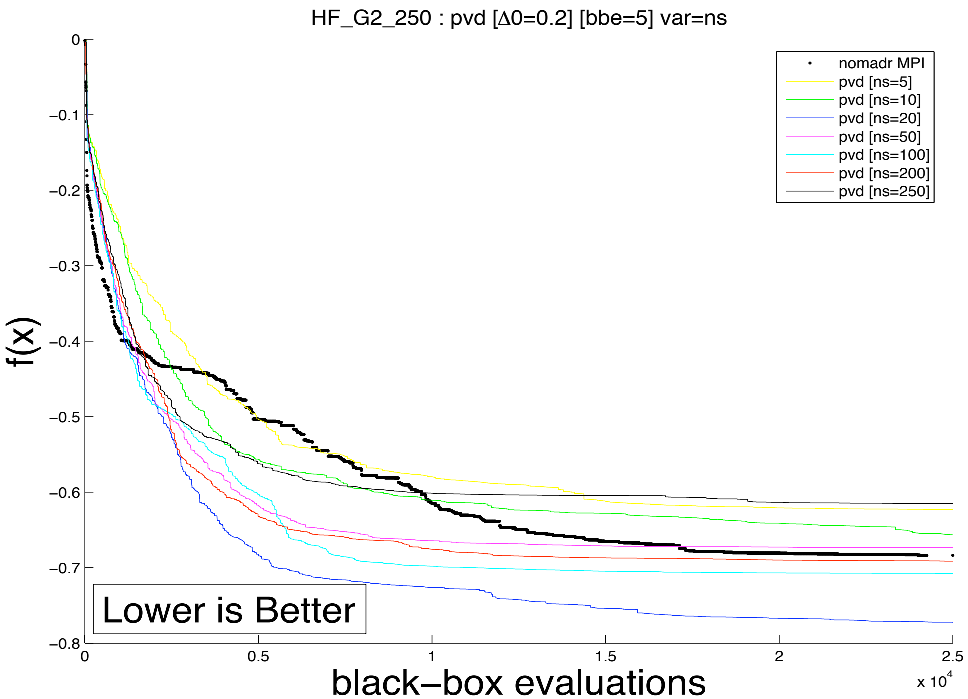
nomadr MPI
pvd [bbe=1]
pvd [bbe=2]
pvd [bbe=3]
pvd [bbe=5]
pvd [bbe=10]
pvd [bbe=20]
pvd [bbe=50]
pvd [bbe=100]
pvd [bbe=500]
Lower is Better
f(x)
black−box evaluations
x 10^4
0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
−0.8
0
0.5
1
1.5
2
2.5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pvd [ns=100]
pvd [ns=200]
pvd [ns=250]
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−0.5
−0.6
−0.7
−0.8
0
0.5
1
1.5
2
2.5

First observations

- Promising preliminary results
- Efficient runs are obtained with small values of parameter *bbe* (best value is a maximum of 5 black-box true evaluations for each subproblem)

Discussion

- New algorithm applying the PVD parallel framework to MADS
- Promising results for large problems
- Convergence results of MADS still hold
- Work in progress:
  - original PVD synchronization $\rightarrow$ new PVD recomposition
  - compare results with APPS (Asynchronous Parallel Pattern Search [Kolda 2005])
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