Personalized shift scheduling problems

From modelling to efficient resolution

GERAD

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Antoine Legrain

And Jérémy Omer, Samuel Rosat, Antoine Gicquel, Flore Caye

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Agenda

1. Introduction
2. Methodology
3. Experiments

Motivation and benchmarks

- Personalized shift-based schedules:
  - Planning horizon;
  - Set of shifts;
  - A demand for each shift and each day;
  - An employee can be assigned to a maximum of one shift each day;
  - Ergonomic constraints;
  - Employee preferences.

- Classical example: Nurse scheduling problem;

Motivation and benchmarks

Schedule from 2023-11-6 to 2023-11-19 | version: 7.8

The status of your schedule is: Success

	Sun 11/4	Sun 11/5	Mon 11/6	Tue 11/7	Wed 11/8	Thu 11/9	Fri 11/10	Sat 11/11	Sun 11/12	Mon 1
Patrick	EARLY	EARLY	**LATE** Nurse	**LATE** Nurse			**EARLY** Nurse	**EARLY** Nurse	**EARLY** HeadNurse	**LAT** Nurs
Andrea	EARLY	LATE	**LATE** HeadNurse		**LATE** HeadNurse	**LATE** HeadNurse	**LATE** Nurse			
Stefaan	EARLY		**EARLY** Nurse	**EARLY** Nurse	**EARLY** Nurse	**EARLY** Nurse				**EAR** Nurs
Sara	LATE			**LATE** HeadNurse	**LATE** Nurse	**LATE** Nurse		**LATE** Nurse	**LATE** Nurse	**LAT** HeadN
Nguyen	NIGHT	NIGHT	**NIGHT** HeadNurse	**NIGHT** HeadNurse			**LATE** HeadNurse	**LATE** HeadNurse	**LATE** HeadNurse	

Motivation and benchmarks

- Classical column generation problems handle hard constraints:
  - Personalized schedules handle many soft constraints.
- Build a dedicated approach for those soft constraints;

- Several benchmarks exist in the literature:
  - INRC (up to 50 nurses over 8 weeks): https://nrpcompetition.kuleuven-kulak.be/
  - INRC-II (up to 120 nurses over 8 weeks): http://www.schedulingbenchmarks.org/nrp/
  - NRP (up to 150 nurses over 1 year): http://www.schedulingbenchmarks.org/nrp/

Nurse Rostering Problem

- ✓ **Nurses:**
  Set of 30 to 120 individualized nurses.
- ✓ **Planning horizon:**
  Number of weeks to schedule: 4 or 8.
- ✓ **Skills:**
  Each nurse has a set of skills among four, e.g. HEADNURSE, CARETAKER, etc. A nurse can use only one skill at a time.
- ✓ **Contracts:**
  The working contracts of each nurse is specified. Several contracts are available, e.g. FULLTIME, PARTTIME, etc.

- ✓ **Work shifts:**
  Working hours of a given day (EARLY, DAY, LATE, NIGHT).
- ✓ **History:**
  The sequence of shifts worked by each nurse before the first day of the planning horizon.
- ✓ **Preferences:**
  For each nurse, the list of shifts he/she had rather not work.
- ✓ **Demand:**
  For each day, shift and skill, the *minimal* and *optimal* numbers of nurses that will be required.

And many constraints

- Number of consecutive shifts;
- Total number of assignments;
- Total number of worked weekends;
- Forbidding/Rewarding some patterns;
- Work the complete weekend or rest;
- …

Challenges

- Personalized schedules;
- Up to 150 nurses on long horizon;
- Classical Resource Constraint Shortest Path takes time:
  - Many soft constraints with lower and upper bounds (no real domination);
  - Long horizon.

- We must develop a specialized algorithm.

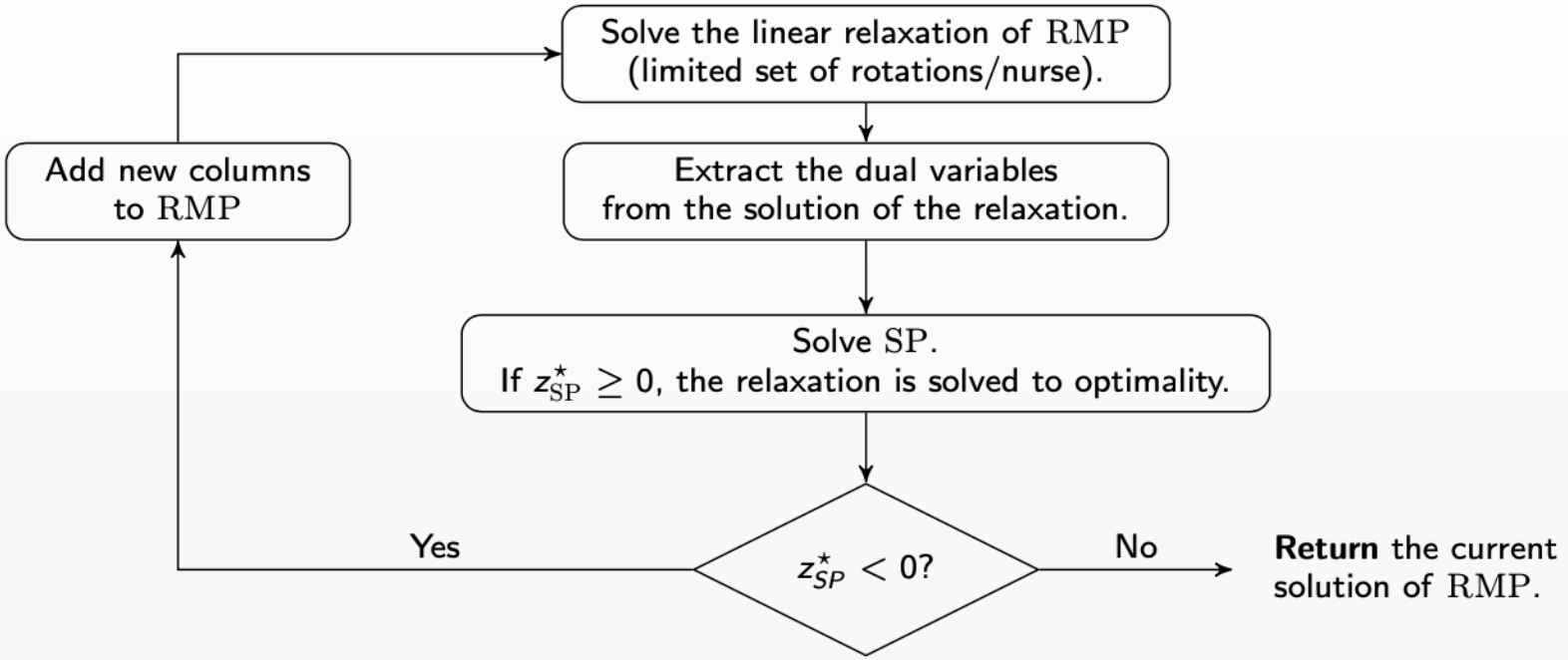
Methodology

- Branch-and-Price:
  - Rotation generation;
  - Roster generation.

- Acceleration techniques:
  - A Soft Resource Constraint Shortest Path.
  - A MIP Heuristic
  - …

Branch-and-Price

- Column generation at every node of the branch-and-bound tree



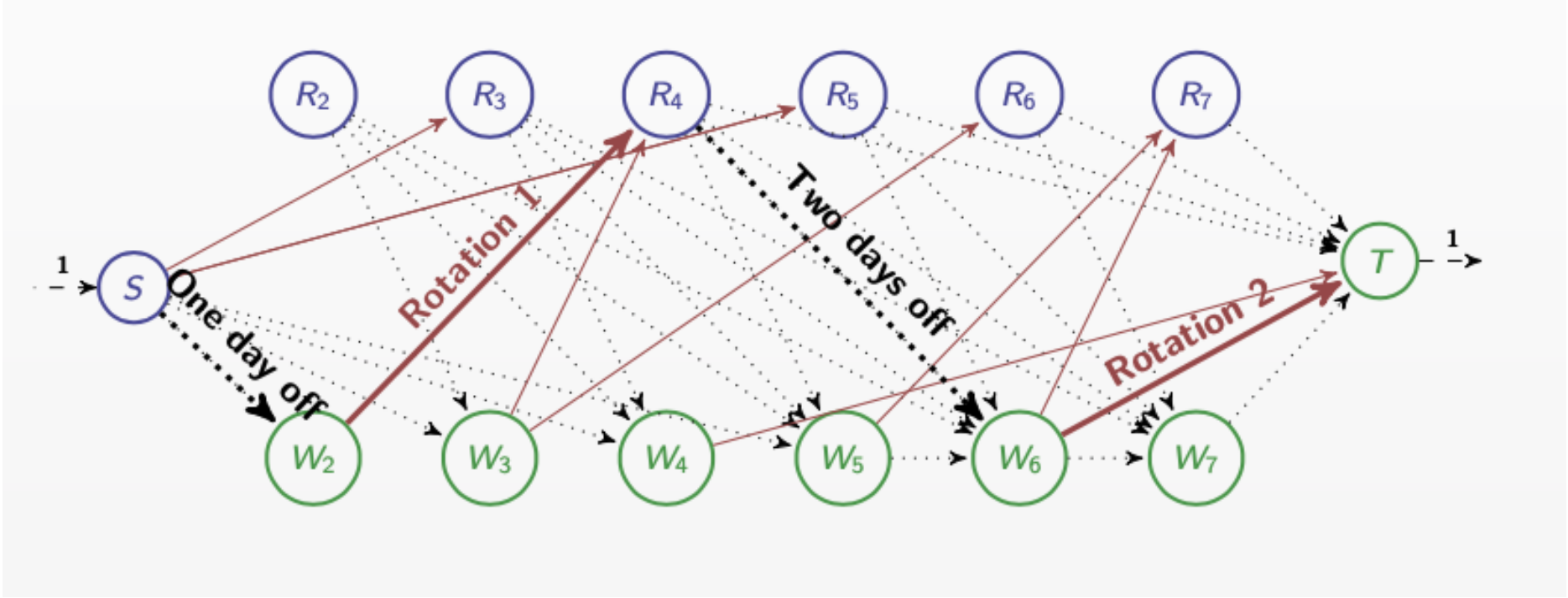
RMP : Restricted Master Problem (limited number of rotations);

SP : Find a rotation of negative reduced cost that is not already in RMP.

Rotation generation

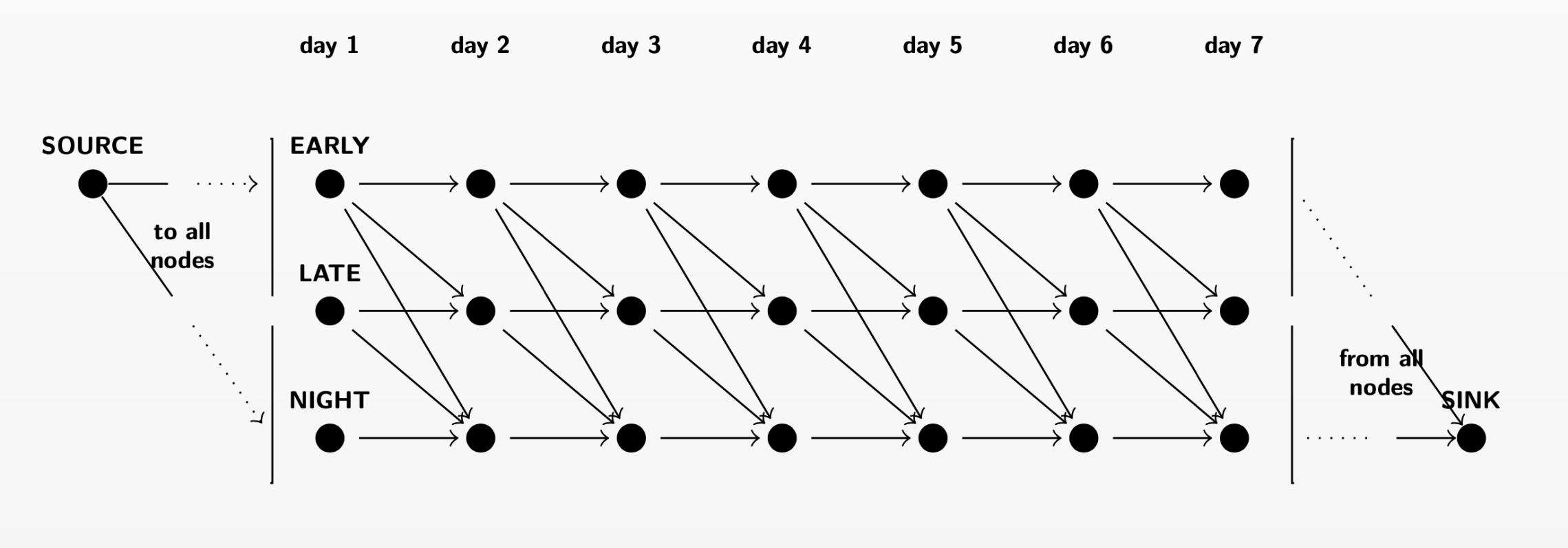
Mon	Tue	Wed	Thu	Fri	Sat	Sun
Off	Early	Day	*Off*	*Off*	Night	Night
	Rotation 1				Rotation 2	

Master : Rotation Graph



Also, some global constraints: e.g., on the total number of assignments and worked week-ends.

Subproblems : Rotation Generation

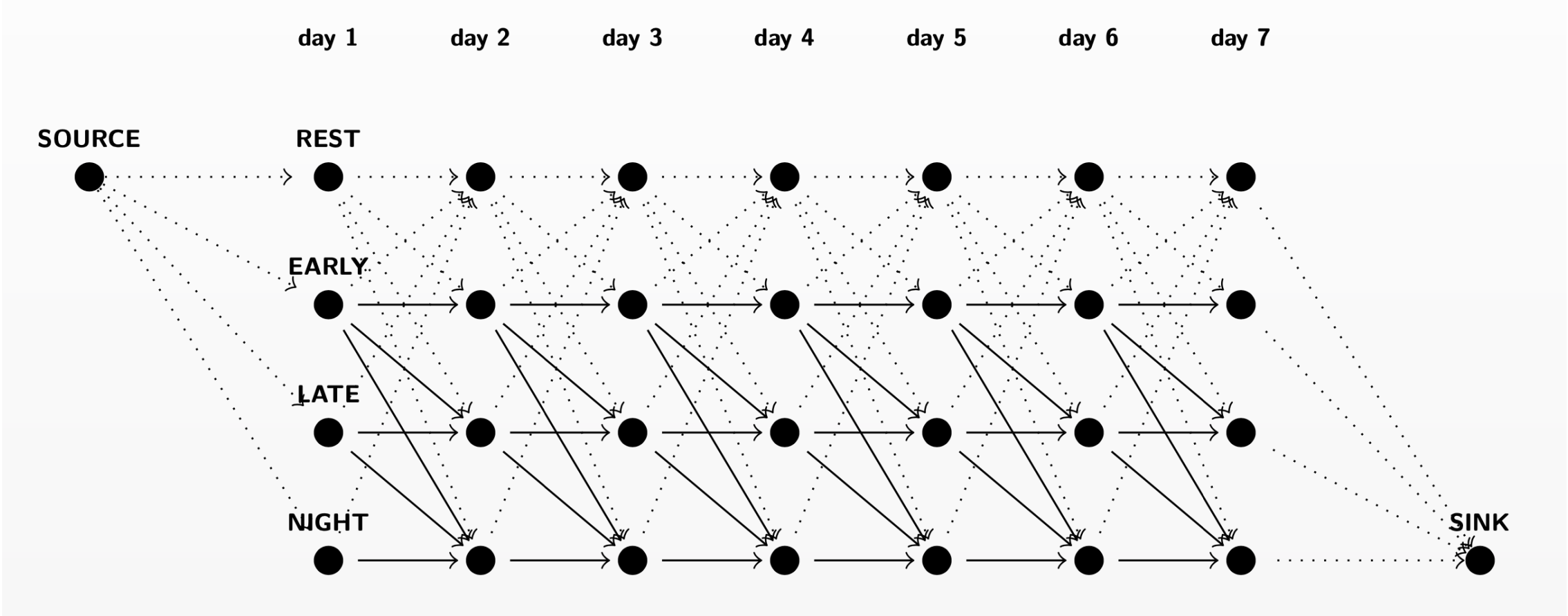


Roster Generation

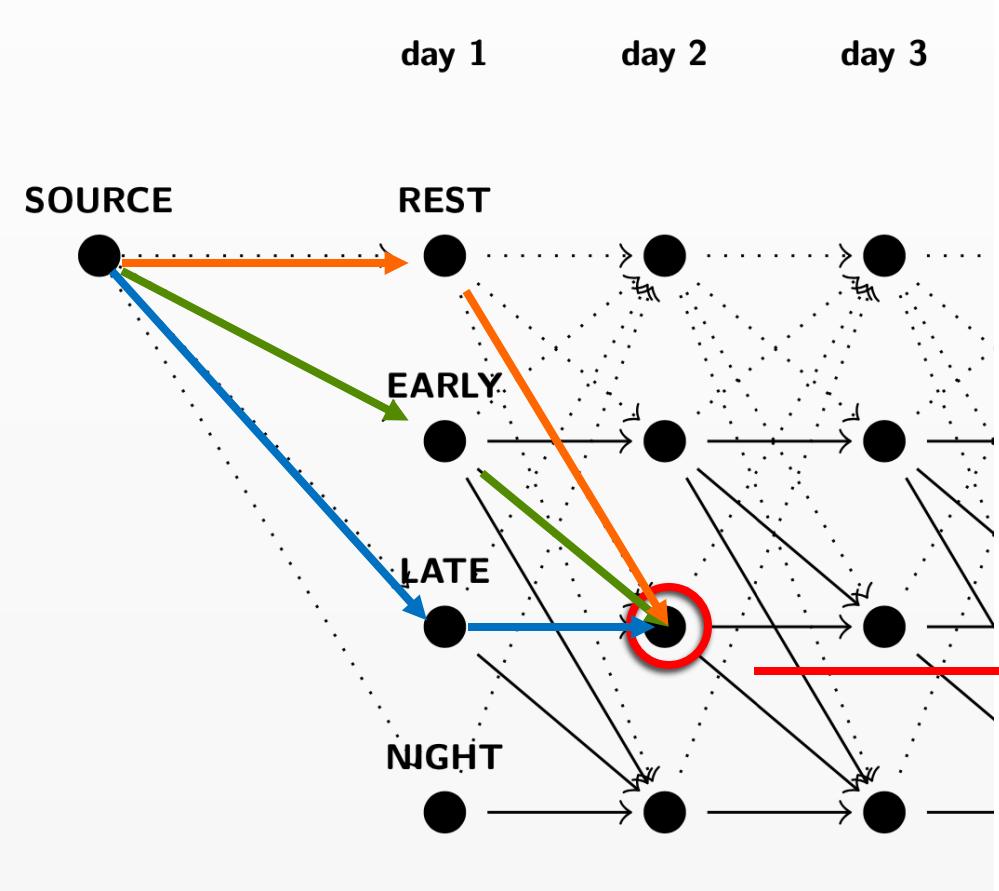
Roster

- **Advantage**: produce a tighter relaxation
- **Disadvantage**: must produce paths over the whole horizon

Roster Generation - Subproblems



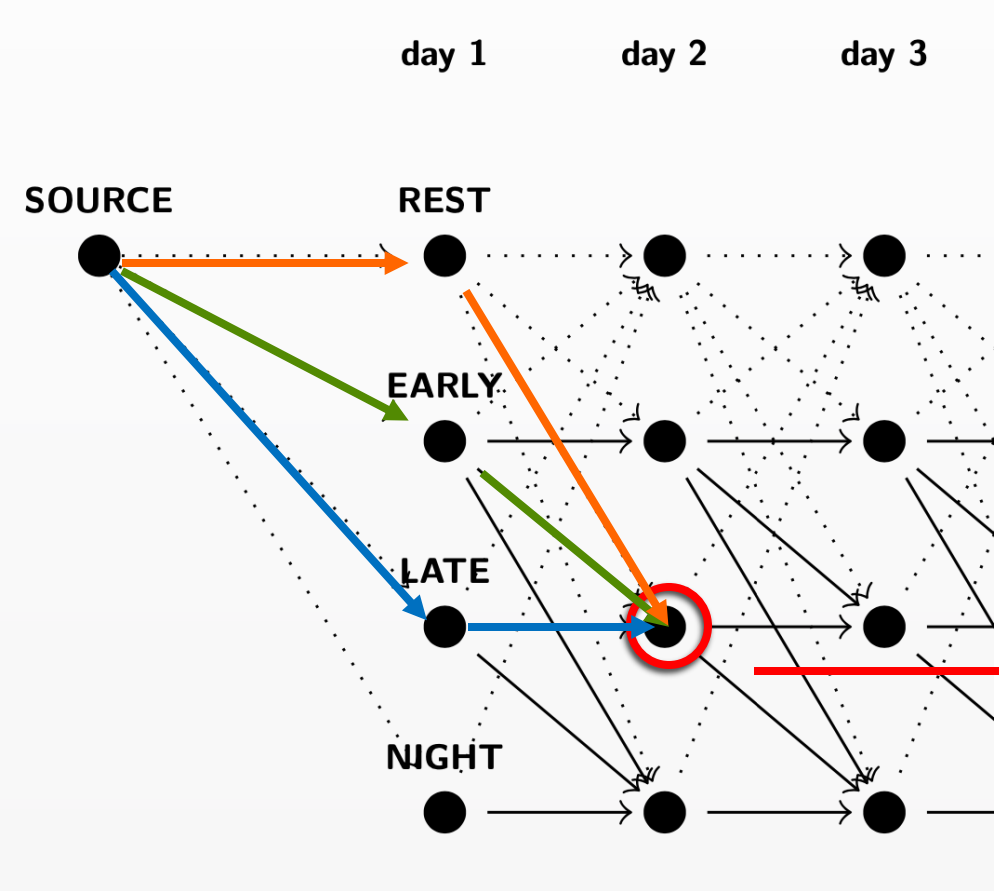
Modeling min/max



- A resource by type of shift you track: rest, work, early etc.
- Lower and upper bounds
- If soft, the cost per unit of violation

- Number of worked shifts: 1 2 2
- Number of consecutive Late shifts: 1 1 2
- etc

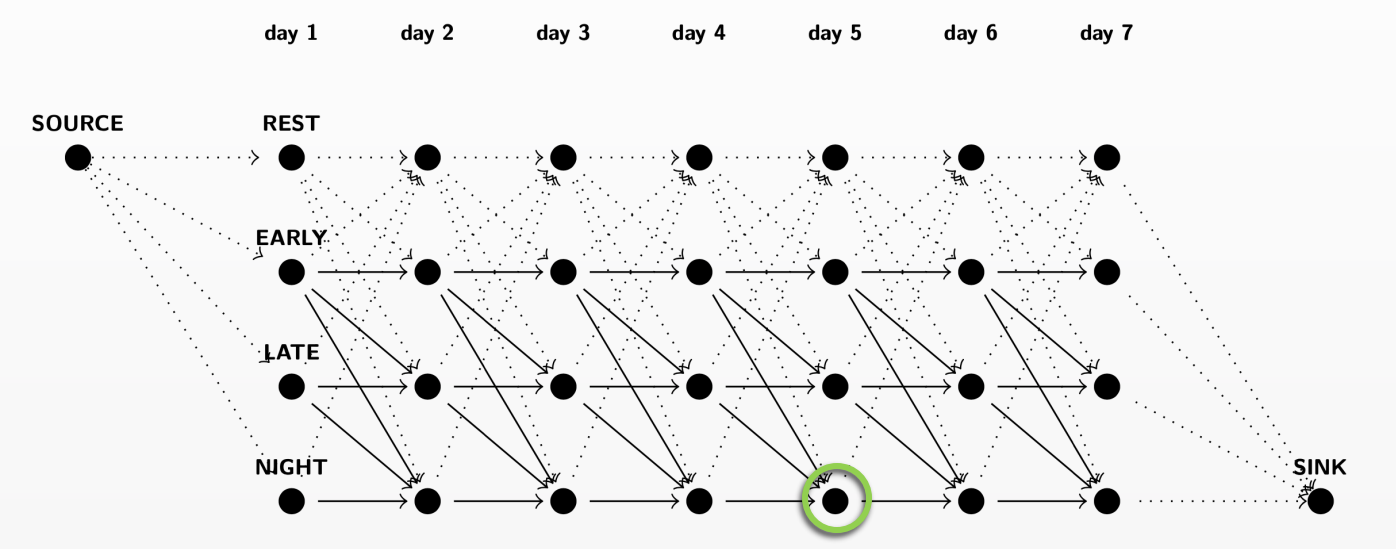
Modeling patterns



- Pattern: (Day, Shift), (Day, Shift) …
- No Friday night if rest the weekend:
  - (Friday, Night), (*, Rest), (*, Rest)
- Avoid Early -> Late -> Early

- Position in the last pattern: 0 2 0

A Soft Resource Constraint Shortest Path



Resource:

1. Consecutive worked days:
   - LB = 3;
   - UB = 5;
   - Penalty = 30.

Partial paths:

1. Cost = -100, Cons worked days = 3
2. Cost = -200, Cons worked days = 1

Classical domination:
Nothing can be said as the resource values are different.

Worst case analysis:

- Stop work immediately.
  Partial paths :
  1. Cost = -100
  2. Cost = -140

- Work 4 days (until partial path 2 reaches the UB).
  Partial paths :
  1. Cost = -40
  2. Cost = -200

A Soft Resource Constraint Shortest Path

- For each resource r, compute the worst case:
  - $w_r = p_r * | l_{1r} - l_{2r} |$
- Compare partial path 1 to partial path 2 for the worst case:
  - If $c_1 + \sum_r w_r \leq c_2$, partial path 1 dominates partial path 2;
  - If $c_2 + \sum_r w_r \leq c_1$, partial path 2 dominates partial path 1;
  - Otherwise, nothing can be said.
- Many more partial paths can be dominated at a given node.

Acceleration - Subproblems

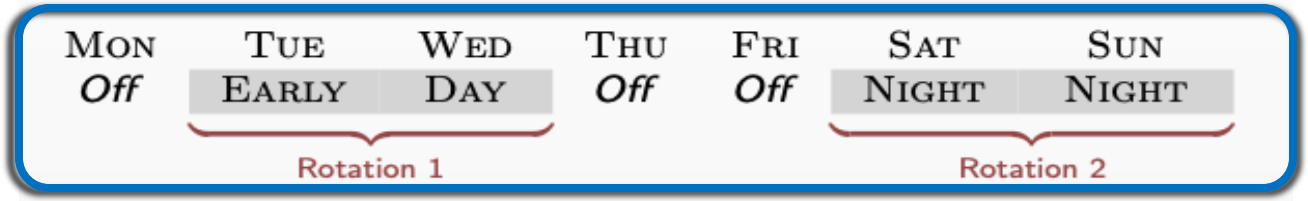
- Many techniques:
  - Decremental state-space relaxation
  - Bidirectional
  - Shortest reverse paths from the sink
  - Truncated labeling
  - Enumerate small paths
  - …

Acceleration – Master problems

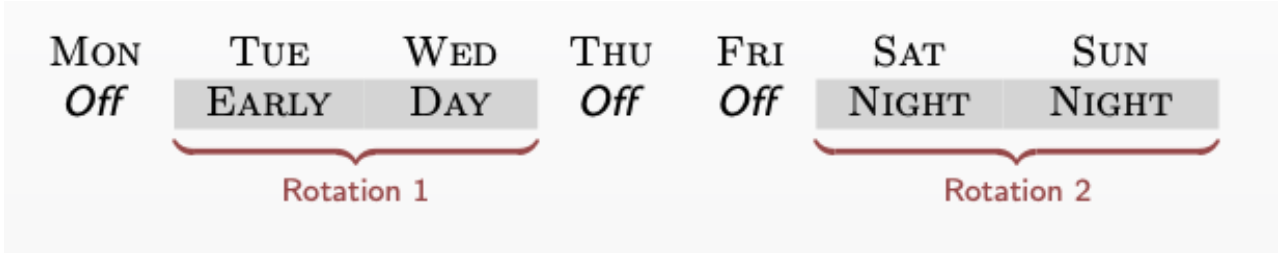
- Many techniques:
  - Strong branching
  - Diving
  - Dual stabilization (tough to tune)
  - Lagrangian lower bound
  - A heuristic based on a MIP

MIP Heuristic

1. Solve Roster decompostion relaxation



2. Cut all generated rosters into rotations



3. Solve the Rotation decomposition with a MIP (with all rotations)

Run in parallel and repeat forever

Experiments

- Run the roster decomposition (in parallel over 8 threads);
- The (open-source) code runs on COIN-OR (BCP and CLP);
- The MIP heuristic uses Gurobi (but can run with CBC);
- Experiments:
  - Features analysis on several instances;
  - Benchmark results:
    - INRC, INRC-II, and NRP instances.

Strategy	Default	ENUM	SRP	ENUM SRP	BIDI	DSSR	RNPL=10	DSSR RNPL=5	DSSR RNPL=10
Instance	Time (s)	Improvement							
n030w4-1	465	4%	58%	2%	55%	37%	36%	33%	33%
n035w4-1	385	18%	54%	0%	50%	51%	32%	36%	29%
n040w4-1	482	30%	58%	-1%	55%	28%	11%	43%	22%
n030w8-1	13772	-1%	65%	1%	61%	34%	14%	20%	28%
n035w8-1	20516	-18%	73%	1%	70%	38%	3%	10%	39%
n040w8-1	16027	13%	67%	-1%	57%	43%	7%	32%	33%
long3	217	0%	24%	-1%	10%	61%	3%	51%	14%
long_late3	3385	-1%	13%	-1%	61%	66%	70%	73%	73%
medium_hidden1	4315	1%	18%	1%	61%	49%	76%	72%	83%
Average		**5%**	**48%**	0%	**53%**	**45%**	**28%**	**41%**	**39%**

Table 1 Computational time improvements for the basic dominance rule

Strategy	Default	ENUM	SRP	ENUM SRP	BIDI	DSSR	RNPL=10	DSSR RNPL=5	DSSR RNPL=10
Instance	Time (s)	Improvement							
n030w4-1	39	-26%	-8%	-13%	8%	-15%	-5%	-5%	-18%
n035w4-1	40	-23%	-20%	-10%	-15%	-5%	-33%	-23%	-33%
n040w4-1	27	-22%	-7%	-11%	-26%	-19%	-52%	-15%	-59%
n030w8-1	544	-21%	-1%	-4%	29%	-2%	-18%	-13%	-5%
n035w8-1	725	-64%	2%	-8%	30%	-6%	-15%	-26%	-22%
n040w8-1	654	-10%	2%	-2%	19%	-20%	-17%	-18%	-36%
long3	83	-1%	-1%	-6%	45%	4%	-118%	-35%	-112%
long_late3	302	0%	-2%	-4%	33%	-10%	-81%	-71%	-86%
medium_hidden1	303	-1%	-7%	-2%	31%	-1%	-61%	-33%	-61%
Average		**-27%**	**-5%**	**-8%**	**7%**	**-11%**	**-23%**	**-16%**	-29%

Table 2 Computational time improvements for the improved dominance rule

Acceleration results – Master problems

Strategy	diving	rotation MIP	strong branching	diving	rotation MIP	strong branching
Instances		LB			UB	
n080w4-2	0.00%	0.01%	0.05%	1.24%	1.39%	0.31%
n110w4-1	0.21%	-0.01%	0.06%	59.48%	59.48%	0.00%
n030w8-1	-0.15%	0.00%	-0.12%	-0.99%	0.74%	-1.23%
n035w8-2	-0.25%	-0.01%	0.03%	54.53%	56.45%	0.77%
n040w8-1	-0.11%	0.03%	0.17%	-1.54%	2.70%	-2.70%
long3	0.00%	0.00%	0.00%	0.41%	0.41%	0.41%
medium_hidden1	0.00%	0.00%	4.70%	13.87%	10.95%	-21.90%
Average	**-0.04%**	**0.00%**	**0.70%**	**18.14%**	**18.88%**	**-3.48%**

Table 3 Assessment of the strong branching strategy and of primal heuristics: improvement of the lower (LB) and upper (UB) bounds compared to the default branch-and-price algorithm; time limit was set to two hours.

INRC

Type	early				late				hidden			
Instance	Time (s)	root LB	LB	UB	Time (s)	root LB	LB	UB	Time (s)	root LB	LB	UB
sprint1	107	56	**56**	**56**	144	37	**37**	**37**	29	31.5	**32**	**32**
sprint2	62	58	**58**	**58**	15	41.4	**42**	**42**	11	32	**32**	**32**
sprint3	145	51	**51**	**51**	36	47.8	**48**	**48**	21	62	**62**	**62**
sprint4	30	58.5	**59**	**59**	107	72.5	**73**	**73**	26	66	**66**	**66**
sprint5	134	57	**58**	**58**	174	43.7	**44**	**44**	90	59	**59**	**59**
sprint6	22	54	**54**	**54**	61	41.5	**42**	**42**	31	129.7	**130**	**130**
sprint7	26	56	**56**	**56**	41	42	**42**	**42**	11	153	**153**	**153**
sprint8	49	56	**56**	**56**	77	17	**17**	**17**	36	204	**204**	**204**
sprint9	103	55	**55**	**55**	87	17	**17**	**17**	267	337.3	**338**	**338**
sprint10	49	52	**52**	**52**	85	42.9	**43**	**43**	29	306	**306**	**306**
medium1	94	240	**240**	**240**	128	156	**157**	**157**	–	95.7	103	120
medium2	26	239.2	**240**	**240**	94	18	**18**	**18**	–	212.4	216	219
medium3	136	235.5	**236**	**236**	155	28.2	**29**	**29**	341	33.4	**34**	**34**
medium4	363	236.2	**237**	**237**	58	34.4	**35**	**35**	4817	75.5	**78**	**78**
medium5	155	302.1	**303**	**303**	450	106.7	**107**	**107**	329	117.2	**118**	**118**
long1	68	197	**197**	**197**	360	235	**235**	**235**	830	345	**346**	**346**
long2	104	218.5	**219**	**219**	336	229	**229**	**229**	170	88.5	**89**	**89**
long3	80	240	**240**	**240**	–	218.5	219	220	3408	37.7	**38**	**38**
long4	61	303	**303**	**303**	138	220.7	**221**	**221**	684	21.8	**22**	**22**
long5	183	284	**284**	**284**	163	82.5	**83**	**83**	1012	41	**41**	**41**
Improvements			**0**	**0**			**1(1)**	**0**			**9(7)**	**0**

Table 4 Results for the INRC-I ; time limit was set to 4 hours.

INRC-II

Instance	Time (s)	root LB	LB	UB	Instance	Time (s)	root LB	LB	UB
n030w4-1	67	1659.5	**1670**	**1670**^{H}	n030w8-1	425	1993.7	**2010**	**2010**^{H}
n030w4-2	377	1809.2	**1815**	**1815**^{B}	n030w8-2	471	1709.4	**1720**	**1720**^{D}
n035w4-1	66	1337.1	**1360**	**1360**^{B}	n035w8-1	-	2407.8	2425	2505^{H}
n035w4-2	196	1075.6	**1080**	**1080**^{H}	n035w8-2	-	2152.2	2205	2270^{H}
n040w4-1	161	1535.2	**1565**	**1565**^{D}	n040w8-1	-	2463.9	2490	2515^{H}
n040w4-2	33	1741.7	**1750**	**1750**^{D}	n040w8-2	-	2284	2305	2330^{H}
n050w4-1	483	1295.5	**1315**	**1315**^{D}	n050w8-1	-	4777.5	4805	4810^{B}
n050w4-2	175	1302.1	**1315**	**1315**^{H}	n050w8-2	7731	4743.3	**4765**	**4765**^{B}
n060w4-1	46	2434.9	**2450**	**2450**^{D}	n060w8-1	-	2099	2120	2150^{D}
n060w4-2	31	2664.3	**2675**	**2675**^{D}	n060w8-2	-	2393.4	2410	2435^{H}
n070w4-1	37	2370.3	**2380**	**2380**^{D}	n070w8-1	-	4474.7	4505	4530^{H}
n070w4-2	69	2105	**2115**	**2115**^{H}	n070w8-2	-	4636.3	4660	4690^{H}
n080w4-1	2811	3292	**3300**	**3300**^{D}	n080w8-1	-	3941.7	3955	4030^{H}
n080w4-2	7514	3177.1	**3185**	**3185**^{H}	n080w8-2	-	4286.9	4295	4335^{H}
n100w4-1	187	1167.2	**1170**	**1170**^{B}	n100w8-1	-	2013.5	2025	2030^{H}
n100w4-2	39	1777.7	**1780**	**1780**^{D}	n100w8-2	-	2128.6	2140	2145^{H}
n110w4-1	816	2321.3	**2330**	**2330**^{D}	n110w8-1	2456	3990	**3990**	**3990**^{B}
n110w4-2	133	2455	**2455**	**2455**^{D}	n110w8-2	-	3440	3440	3460^{H}
n120w4-1	11351	2011.5	**2020**	**2020**^{H}	n120w8-1	-	2440	2440	2450^{D}
n120w4-2	70	2045.7	**2050**	**2050**^{D}	n120w8-2	14162	2871.7	**2875**	**2875**^{B}
Improvements			**20**	**20(20)**				**5**	**5(5)**

Table 5 **Results for the static INRC–II; time limit was set to one hour per week.**

NRP

				best published		our roster-based approach			
instance	**weeks**	**nurses**	**shifts**	**LB**	**UB**	**time (s)**	**root LB**	**LB**	**UB**
Instance1	2	8	1	**607**[1]	**607**[1]	2	558	**607**	**607**
Instance2	2	14	2	**828**[1]	**828**[1]	0	828	**828**	**828**
Instance3	2	20	3	**1001**[1]	**1001**[1]	1	1001	**1001**	**1001**
Instance4	4	10	2	**1716**[1]	**1716**[1]	2	1716	**1716**	**1716**
Instance5	4	16	2	**1143**[1]	**1143**[1]	9	1140.6	**1143**	**1143**
Instance6	4	18	3	**1950**[1]	**1950**[1]	9	1949	**1950**	**1950**
Instance7	4	20	3	**1056**[1]	**1056**[1]	112	1054.1	**1056**	**1056**
Instance8	4	30	4	**1300**[1]	**1300**[1]	-	1296.6	1298	**1300**
Instance9	4	36	4	**439**[1]	**439**[1]	-	405.7	406	**439**
Instance10	4	40	5	**4631**[1]	**4631**[1]	100	4631	**4631**	**4631**
Instance11	4	50	6	**3443**[1]	**3443**[1]	21	3443	**3443**	**3443**
Instance12	4	60	10	**4040**[1]	**4040**[1]	4131	4040	**4040**	**4040**
Instance13	4	120	18	1348[1]	1356[2]	-	1301	1301	1351
Instance14	6	32	4	**1278**[1]	**1278**[1]	4830	1278	**1278**	**1278**
Instance15	6	45	6	3820[1]	3853[1]	-	-	-	3879
Instance16	8	20	3	**3225**[1]	**3225**[1]	63	3223.5	**3225**	**3225**
Instance17	8	32	4	**5746**[1]	**5746**[1]	669	5746	**5746**	**5746**
Instance18	12	22	3	4404[1]	4459[1]	-	4417.3	4421	4459
Instance19	12	40	5	3144[1]	3204[1]	-	-	-	3174
Instance20	26	50	6	4765[1]	4913[1]	36089	4769	**4769**	**4769**
Instance21	26	100	8	21122[1]	21402[2]	-	-	-	-
Instance22	52	50	10	-	32126[2]	-	-	-	-
Instance23	52	100	16	-	19704[2]	-	-	-	-
Instance24	52	150	32	-	58480[2]	-	-	-	-
Improvements								**2(1)**	**3(1)**

Table 6 Results for the NRP instances: indices 1 and 2 refer to solutions published respectively in (Smet 2018) (a flow-based MIP) and (Strandmark et al. 2020) (a column generation-based heuristic); time limit was set to one hour per week.

Take home message

- Soft domination has sped up the resolution time by 25 times for an 8-weeks roster;
- The MIP heuristic produces good integer solution.
- The roster approach :
  - is computationally manageable;
  - has a very tight lower bound;
  - proves optimality for many instances of several benchmarks in a very small time.
- Ref: Legrain A & Omer J., *"A Dedicated Pricing Algorithm to Solve a Large Family of Nurse Scheduling Problems with Branch-and-Price", Informs Journal On Computing* (2023)

Reference & Open source softwares

Cahier GERAD

Scheduler

Webapp

INRC-II with 24 hours

	Time limit: 8 hours				Time limit: 1 day			
Instance	Time (s)	root LB	LB	UB	Time (s)	LB	UB	LNS UB
n030w8-1	425	1993.7	**2010**	**2010^{H}**				2025
n030w8-2	471	1709.4	**1720**	**1720^{D}**				1740
n035w8-1	-	2407.8	2425	2505^{H}	-	2430	2460^{H}	2475
n035w8-2	-	2152.2	2205	2270^{H}	-	2215	2265^{H}	2265
n040w8-1	-	2463.9	2490	2515^{H}	-	2495	2515^{B}	2605
n040w8-2	-	2284	2305	2330^{H}	-	2310	2325^{D}	2320
n050w8-1	-	4777.5	4805	4810^{B}	-	4805	4810^{H}	4830
n050w8-2	7731	4743.3	**4765**	**4765^{B}**				4775
n060w8-1	-	2099	2120	2150^{D}	-	2125	2145^{H}	2200
n060w8-2	-	2393.4	2410	2435^{H}	-	2415	2430^{H}	2475
n070w8-1	-	4474.7	4505	4530^{H}	69115	**4520**	**4520^{B}**	4555
n070w8-2	-	4636.3	4660	4690^{H}	20312	**4675**	**4675^{H}**	4700
n080w8-1	-	3941.7	3955	4030^{H}	-	3955	4055^{H}	4110
n080w8-2	-	4286.9	4295	4335^{H}	-	4295	4325^{H}	4335
n100w8-1	-	2013.5	2025	2030^{H}	-	2025	2030^{H}	2065
n100w8-2	-	2128.6	2140	2145^{H}	-	2140	2145^{H}	2190
n110w8-1	2456	3990	**3990**	**1315^{B}**				4000
n110w8-2	-	3440	3440	3460^{H}	11430	**3440**	**3440^{B}**	3440
n120w8-1	-	2440	2440	2450^{D}	-	2440	2450^{B}	2450
n120w8-2	14162	2871.7	**2875**	**2875^{B}**				2875

Table 8 Results for the static INRC–II over 8 weeks with time limit set to 8 and 24 hours.

INRC-II with CBC

Instance	Time (s)	LB	UB	Instance	Time (s)	LB	UB
n030w4-1	86	**1670**	**1670**	n030w8-1	918	**2010**	**2010**
n030w4-2	1425	**1815**	**1815**	n030w8-2	4774	**1720**	**1720**
n035w4-1	71	**1360**	**1360**	n035w8-1	28802	2425	2585
n035w4-2	81	**1080**	**1080**	n035w8-2	28824	2195	2365
n040w4-1	170	**1565**	**1565**	n040w8-1	28800	2490	2565
n040w4-2	36	**1750**	**1750**	n040w8-2	28847	2305	2435
n050w4-1	1076	**1315**	**1315**	n050w8-1	28804	4800	4835
n050w4-2	206	**1315**	**1315**	n050w8-2	10627	**4765**	**4765**
n060w4-1	42	**2450**	**2450**	n060w8-1	28825	2115	2275
n060w4-2	31	**2675**	**2675**	n060w8-2	28817	2410	2595
n070w4-1	37	**2380**	**2380**	n070w8-1	28802	4505	4540
n070w4-2	75	**2115**	**2115**	n070w8-2	28803	4660	4730
n080w4-1	7806	**3300**	**3300**	n080w8-1	28861	3950	4105
n080w4-2	14400	3185	3190	n080w8-2	28844	4295	4440
n100w4-1	138	**1170**	**1170**	n100w8-1	28866	2025	2080
n100w4-2	83	**1780**	**1780**	n100w8-2	28840	2140	2215
n110w4-1	637	**2330**	**2330**	n110w8-1	10108	**3990**	**3990**
n110w4-2	59	**2455**	**2455**	n110w8-2	28800	3440	3490
n120w4-1	14288	**2020**	**2020**	n120w8-1	28820	2440	2450
n120w4-2	67	**2050**	**2050**	n120w8-2	28819	2875	2895

Table 9 **Results for the INRC–II using COIN-OR CBC solver; time limit was set to one hour per week.**

INRC-II Rotations vs Rosters

Instance	Rotations		Rosters							
	LB	UB	Root LB	Increase	LB	Increase	UB	Decrease	CBC UB	Decrease
n030w4-1	1615	1685	1659.5	2.76%	1670	3.41%	1670	0.89%	1670	0.89%
n030w4-2	1740	1840	1809.2	3.98%	1815	4.31%	1815	1.36%	1815	1.36%
n035w4-1	1250	1415	1337.1	6.97%	1360	8.80%	1360	3.89%	1360	3.89%
n035w4-2	1045	1145	1075.6	2.93%	1080	3.35%	1080	5.68%	1080	5.68%
n040w4-1	1335	1640	1535.2	15.00%	1565	17.23%	1565	4.57%	1565	4.57%
n040w4-2	1570	1865	1741.7	10.94%	1750	11.46%	1750	6.17%	1750	6.17%
n050w4-1	1195	1445	1295.5	8.41%	1315	10.04%	1315	9.00%	1315	9.00%
n050w4-2	1200	1405	1302.1	8.51%	1315	9.58%	1315	6.41%	1315	6.41%
n060w4-1	2380	2465	2434.9	2.31%	2450	2.94%	2450	0.61%	2450	0.61%
n060w4-2	2615	2730	2664.3	1.89%	2675	2.29%	2675	2.01%	2675	2.01%
n070w4-1	2280	2430	2370.3	3.96%	2380	4.39%	2380	2.06%	2380	2.06%
n070w4-2	1990	2125	2105	5.78%	2115	6.28%	2115	0.47%	2115	0.47%
n080w4-1	3140	3320	3292	4.84%	3300	5.10%	3300	0.60%	3300	0.60%
n080w4-2	3045	3240	3177.1	4.34%	3185	4.60%	3185	1.70%	3190	1.54%
n100w4-1	1055	1230	1167.2	10.64%	1170	10.90%	1170	4.88%	1170	4.88%
n100w4-2	1470	1855	1777.7	20.93%	1780	21.09%	1780	4.04%	1780	4.04%
n110w4-1	2210	2390	2321.3	5.04%	2330	5.43%	2330	2.51%	2330	2.51%
n110w4-2	2255	2525	2455	8.87%	2455	8.87%	2455	2.77%	2455	2.77%
n120w4-1	1790	2165	2011.5	12.37%	2020	12.85%	2020	6.70%	2020	6.70%
n120w4-2	1820	2220	2045.7	12.40%	2050	12.64%	2050	7.66%	2050	7.66%
	Average			**7.64%**		**8.28%**		**3.70%**		**3.69%**

Table 10 **Comparison of the rotation and roster formulations over INRC–II 4-weeks instances; time limit was set to 4 hours.**

INRC-II Rotations vs Rosters

Instance	Rotations		Rosters (over 8 hours)							
	LB	UB	Root LB	Increase	LB	Increase	UB	Decrease	CBC UB	Decrease
n030w8-1	1920	2070	1993.7	3.84%	2010	4.69%	2010	2.90%	2010	2.90%
n030w8-2	1620	1735	1709.4	5.52%	1720	6.17%	1720	0.86%	1720	0.86%
n035w8-1	2330	2555	2407.8	3.34%	2425	4.08%	2505	1.96%	2585	-1.17%
n035w8-2	2180	2305	2152.2	-1.28%	2205	1.15%	2270	1.52%	2365	-2.60%
n040w8-1	2340	2620	2463.9	5.29%	2490	6.41%	2515	4.01%	2565	2.10%
n040w8-2	2205	2420	2284	3.58%	2305	4.54%	2330	3.72%	2435	-0.62%
n050w8-1	4625	4900	4777.5	3.30%	4805	3.89%	4810	1.84%	4835	1.33%
n050w8-2	4530	4925	4743.3	4.71%	4765	5.19%	4765	3.25%	4765	3.25%
n060w8-1	1970	2345	2099	6.55%	2120	7.61%	2150	8.32%	2275	2.99%
n060w8-2	2260	2590	2393.4	5.90%	2410	6.64%	2435	5.98%	2595	-0.19%
n070w8-1	4400	4595	4474.7	1.70%	4505	2.39%	4530	1.41%	4540	1.20%
n070w8-2	4540	4760	4636.3	2.12%	4660	2.64%	4690	1.47%	4730	0.63%
n080w8-1	3775	4180	3941.7	4.42%	3955	4.77%	4030	3.59%	4105	1.79%
n080w8-2	4125	4450	4286.9	3.92%	4295	4.12%	4335	2.58%	4440	0.22%
n100w8-1	2005	2125	2013.5	0.42%	2025	1.00%	2030	4.47%	2080	2.12%
n100w8-2	2125	2210	2128.6	0.17%	2140	0.71%	2145	2.94%	2215	-0.23%
n110w8-1	3870	4010	3990	3.10%	3990	3.10%	3990	0.50%	3990	0.50%
n110w8-2	3375	3560	3440	1.93%	3440	1.93%	3460	2.81%	3490	1.97%
n120w8-1	2295	2600	2440	6.32%	2440	6.32%	2450	5.77%	2450	5.77%
n120w8-2	2535	3095	2871.7	13.28%	2875	13.41%	2875	7.11%	2895	6.46%
		Average		**3.91%**		**4.54%**		**3.35%**		**1.46%**

Table 11 **Comparison of the rotation and roster formulations over INRC–II 8-weeks instances; time limit was set to 8 hours.**