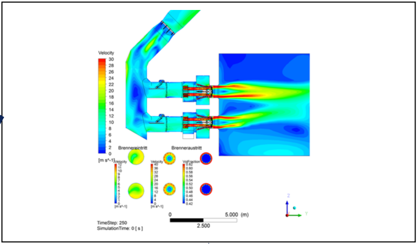
Polytechnique Montreal
Department of Mathematics and Industrial Engineering

Advances in direct search methods for multiobjective derivative-free optimization

Ludovic SALOMON

Motivation: Dimensioning a solar plant



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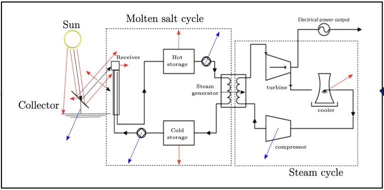

Figure: A solar power plant (source) and its schematic representation (taken from [Lemyre Garneau, 2015]).

Motivation: Dimensioning a solar plant

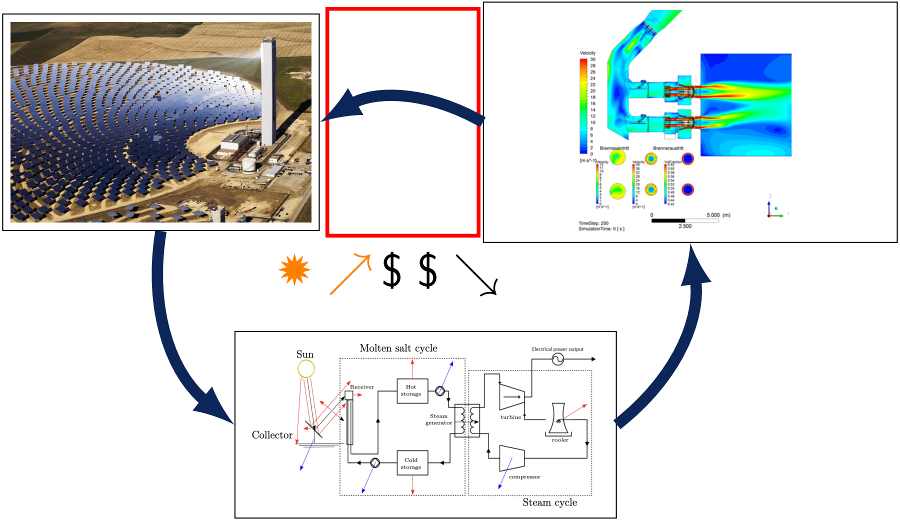

Figure: A solar power plant (source) and its schematic representation (taken from [Lemyre Garneau, 2015]).

The problem

$\Lsh$ **Objective function**

Minimize $f(x)$

subject to $\Lsh$ **Feasible decision space Ω**

$x \in \Omega \subseteq \mathbb{R}^n$

$\hookrightarrow$ **Decision vector**

The problem

$$\begin{array}{ll} \text{Minimize} & f(x) = (f_1(x), f_2(x), \ldots, f_m(x))^\top \\ \text{subject to} & \\ & x \in \Omega \subseteq \mathbb{R}^n \end{array}$$

- $f_i : \mathbb{R}^n \to \mathbb{R} \cup \{\infty\}$ for $i = 1, 2, \ldots, m$, $m \geq 2$, are *objective functions*.
- $\mathbb{R}^m$ is the **objective space**.

The problem

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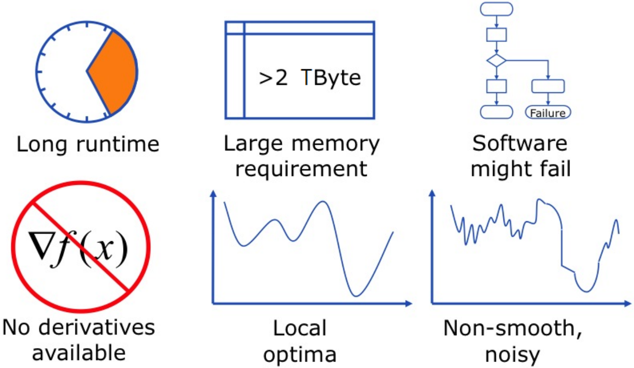
- $f_i : \mathbb{R}^n \to \mathbb{R} \cup \{\infty\}$ for $i = 1, 2, \ldots, m$, $m \geq 2$, are *objective functions*.
- $\mathbb{R}^m$ is the **objective space**.
- $\Omega = \{x \in \mathcal{X} : c_j(x) \leq 0, \ \forall j \in \mathcal{J}\} \subset \mathbb{R}^n$.
- $\mathcal{X}$ is the **set of unrelaxable constraints**.
- $c_j : \mathbb{R}^n \to \mathbb{R} \cup \{\infty\}$ for $j \in \mathcal{J}$ are **relaxable constraints**.

The f_i for $i = 1, 2, \ldots, m$ and c_j for $j \in \mathcal{J}$, are supposed to be blackboxes.

Derivative-free optimization and blackbox optimization



Derivative-free optimization and blackbox optimization

Definition (Taken from [Audet and Hare, 2017])

"**Derivative-free optimization** is the mathematical study of optimization algorithms that do not use derivatives."

Definition (Taken from [Audet and Hare, 2017])

"**Blackbox optimization** is the study of design and analysis of algorithms that assume the objective and/or constraint functions are given by blackboxes."

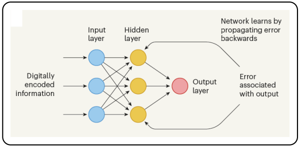
Another application of (multiobjective) blackbox optimization

- Tuning of hyperparameters of neural networks.

Architecture parameters: nb of layers, dropout, ...

Algorithm choice (Adam, SGD,...)

Learning rate



Loss objective value

Training time value

Figure: Neural network illustration taken from `https://www.nature.com/articles/d41586-024-02392-8`.

Pareto dominance

Definition (notations taken from [Audet et al., 2008])

Given two decision vectors x^1 and x^2 in Ω,

- $x^1 \preccurlyeq x^2$ (x^1 *weakly dominates* x^2) if and only if $f_i(x^1) \leq f_i(x^2)$ for $i = 1, 2, \ldots, m$.
- $x^1 \prec x^2$ (x^1 *dominates* x^2) if and only if $x^1 \preccurlyeq x^2$ and at least one objective is strictly better than another.
- $x^1 \prec\prec x^2$ (x^1 *strictly dominates* x^2) if and only if $f_i(x^1) < f_i(x^2)$ for $i = 1, 2, \ldots, m$.
- $x^1 \sim x^2$ (x^1 and x^2 are *incomparable*) if neither x^1 weakly dominates x^2 nor x^2 weakly dominates x^1.

Remark: Definitions can be extended to objective vectors.

Pareto dominance

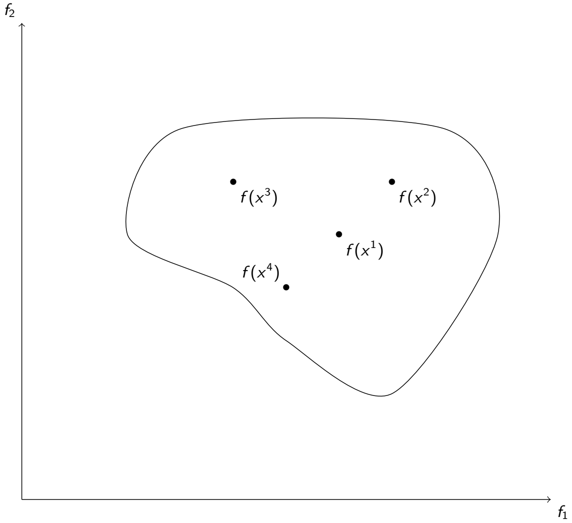
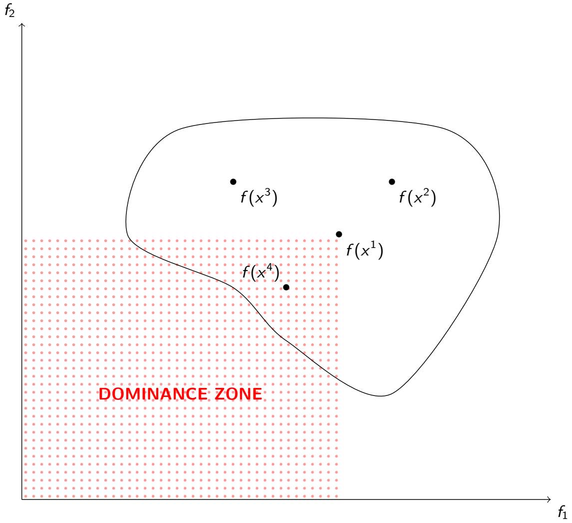


Figure: An illustration of Pareto dominance for a minimization biobjective problem.

Pareto dominance



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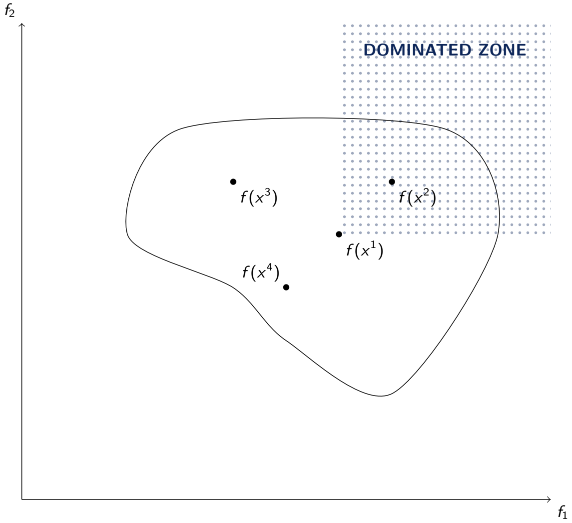

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Pareto dominance

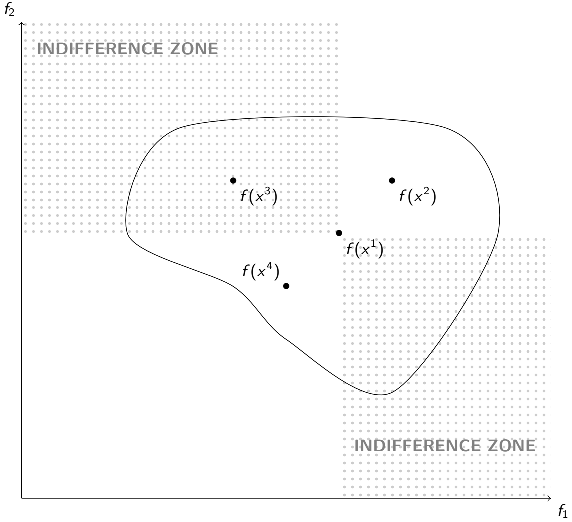


Figure: An illustration of Pareto dominance for a minimization biobjective problem.

Pareto front and Pareto set

Definition

$x \in \Omega$ is said to be Pareto-optimal if there is not other vector in Ω that dominates it. The set of Pareto-optimal solutions (decision variables) is called the Pareto set and the image of the Pareto set is called the Pareto front.

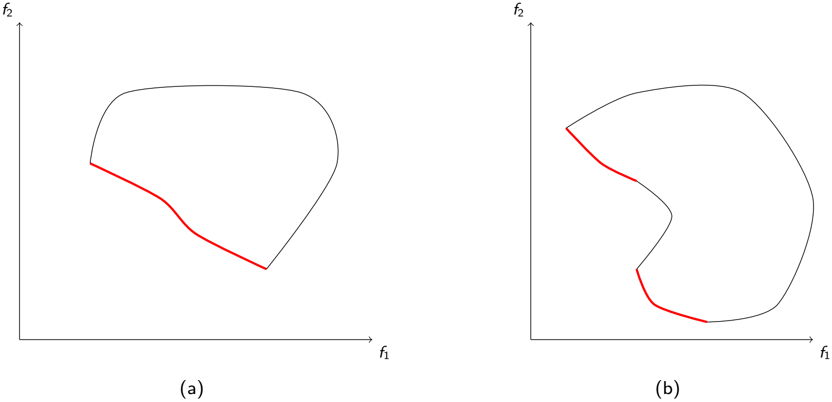
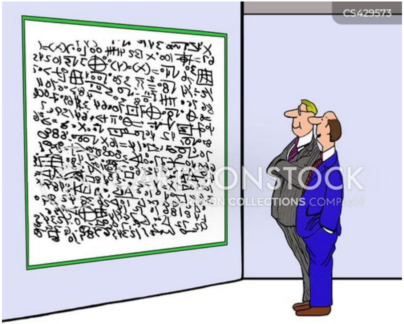


Figure: From left to right: (a) A non-convex Pareto front for a biobjective minimization problem. (b) A piecewise continuous Pareto front for a biobjective minimization problem.

Last remarks



"When you put it like that, it makes complete sense."

Pros

A more precise modeling.

But

Cons

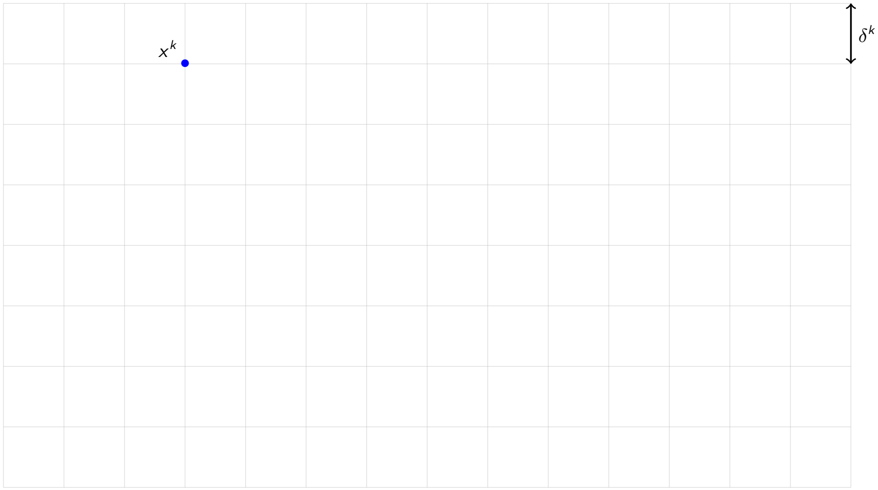
(Generally) harder to solve than single-objective optimization problems.

How ? Direct search methods

Direct search methods for single-objective optimization

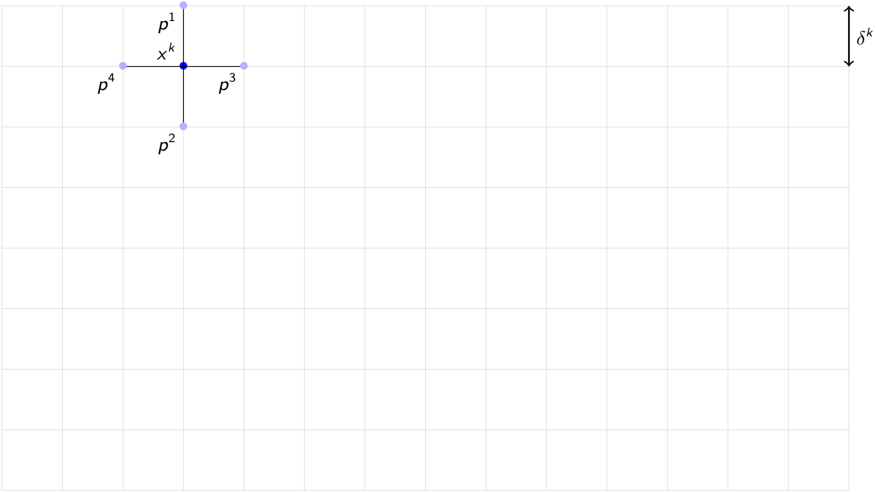
- Coordinate Search (CS) [Fermi and Metropolis, 1952].
- Nelder-Mead (NM) [Nelder and Mead, 1965].
- Mesh Adaptive Direct Search (MADS) [Audet and Dennis, 2006].
- Generated Set Search (GSS) [Kolda et al., 2003].

Coordinate search (CS) for single objective optimization



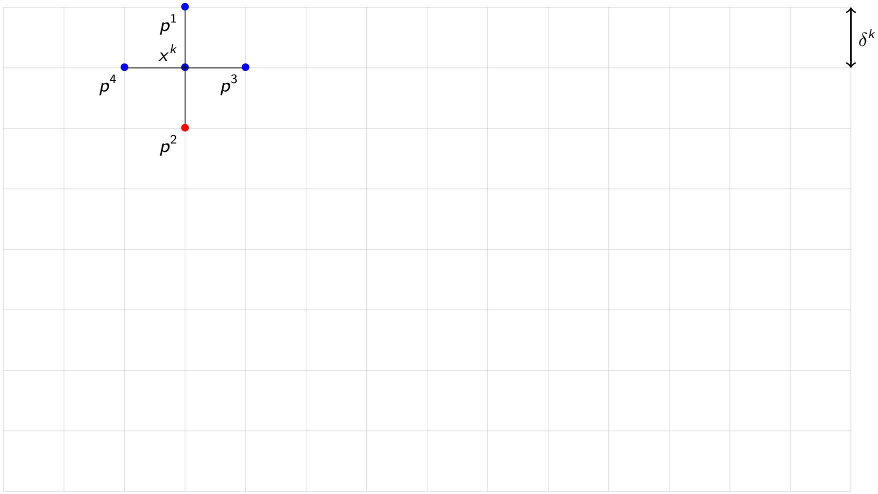
INITIAL INCUMBENT

Coordinate search (CS) for single objective optimization



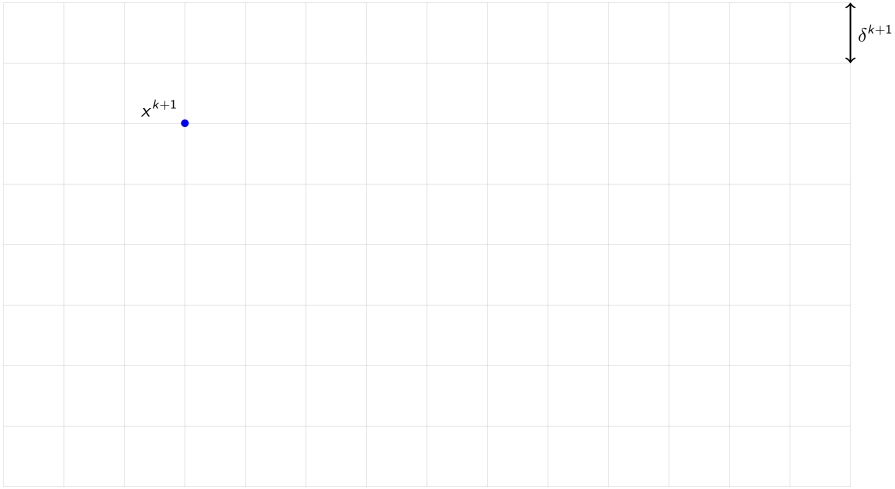
POLL: GENERATION

Coordinate search (CS) for single objective optimization



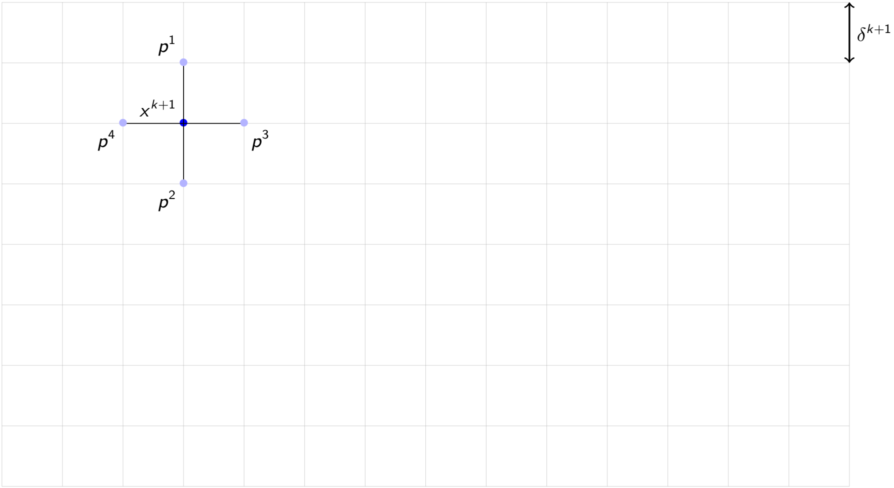
POLL: EVALUATION

Coordinate search (CS) for single objective optimization



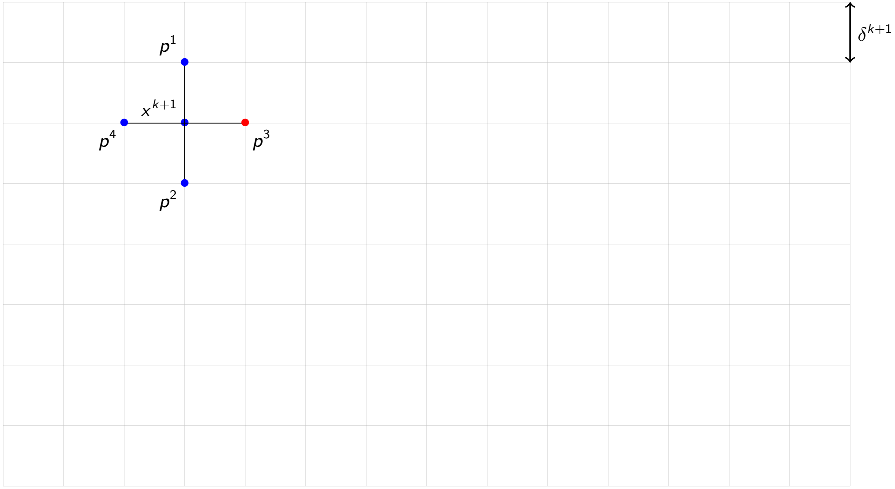
SUCCESS: UPDATE $\delta^{k+1} = \delta^k$

Coordinate search (CS) for single objective optimization



POLL: GENERATION

Coordinate search (CS) for single objective optimization

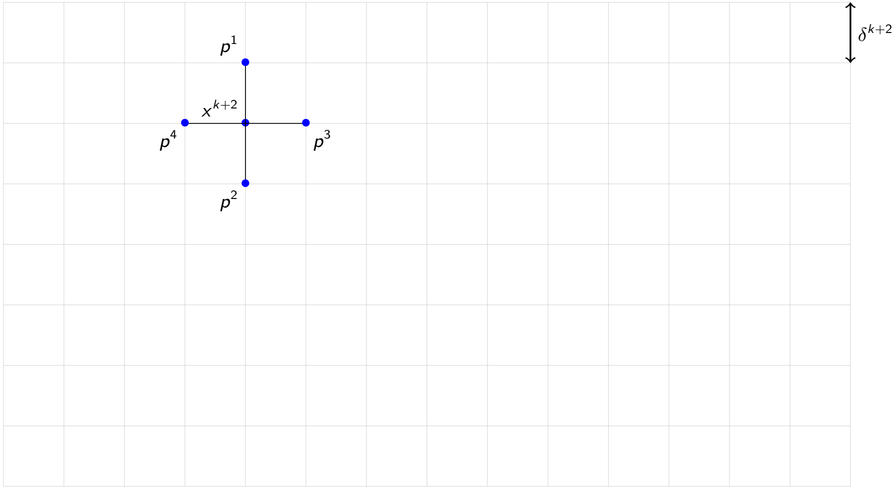


POLL: EVALUATION

Coordinate search (CS) for single objective optimization

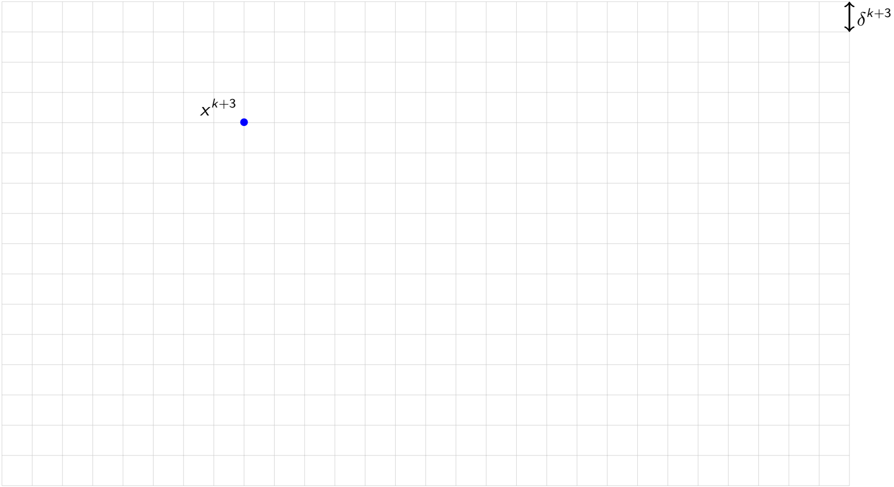
SUCCESS: UPDATE $\delta^{k+2} = \delta^{k+1}$

Coordinate search (CS) for single objective optimization



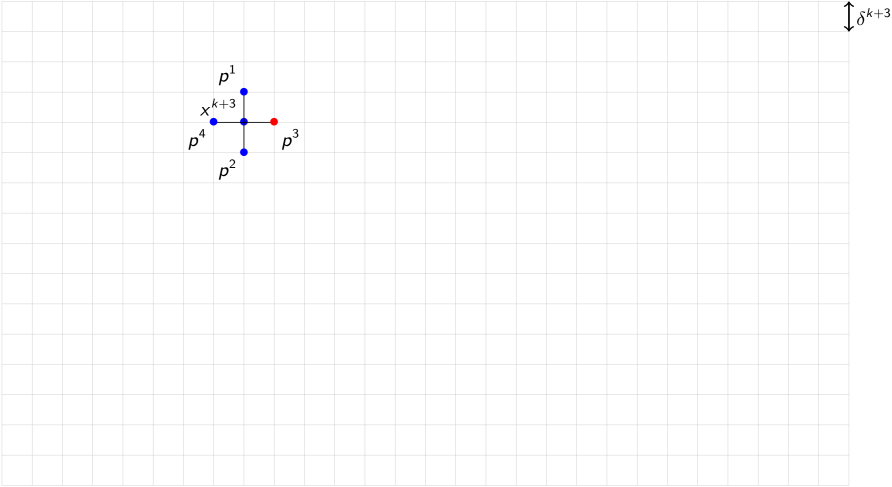
POLL: GENERATION AND EVALUATION

Coordinate search (CS) for single objective optimization



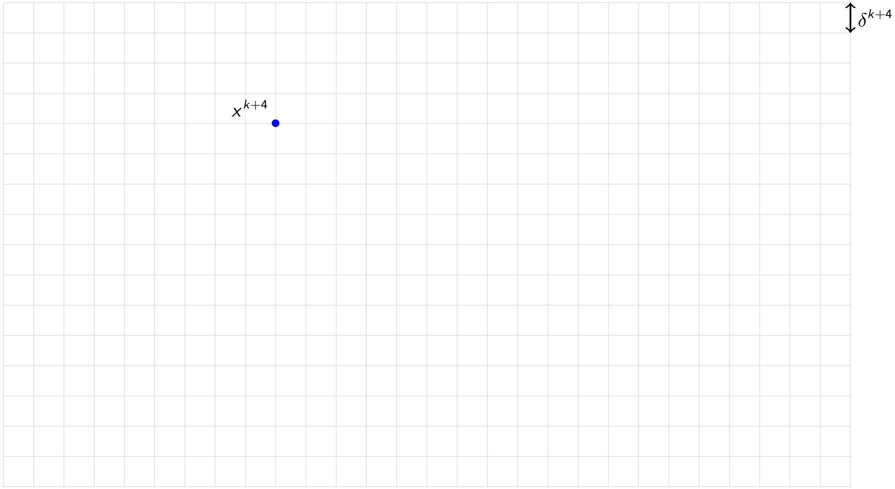
FAILURE: UPDATE $\delta^{k+3} = (1/2)\delta^{k+2}$

Coordinate search (CS) for single objective optimization



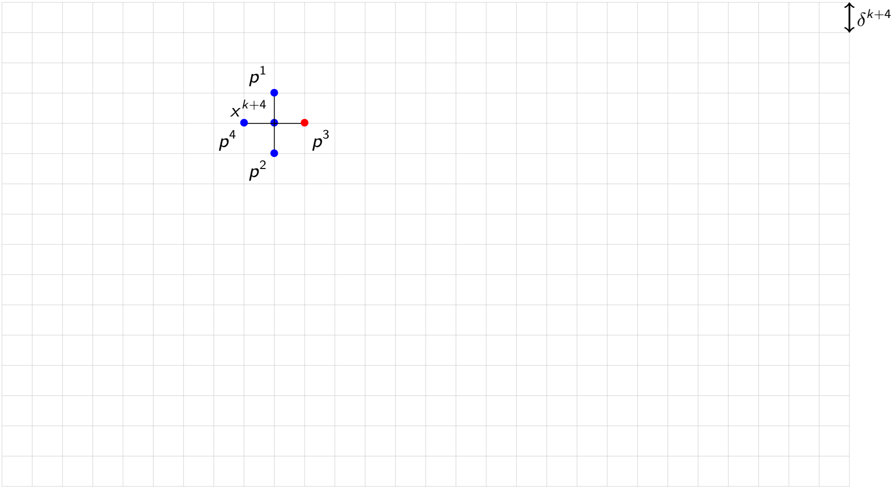
POLL: GENERATION AND EVALUATION

Coordinate search (CS) for single objective optimization



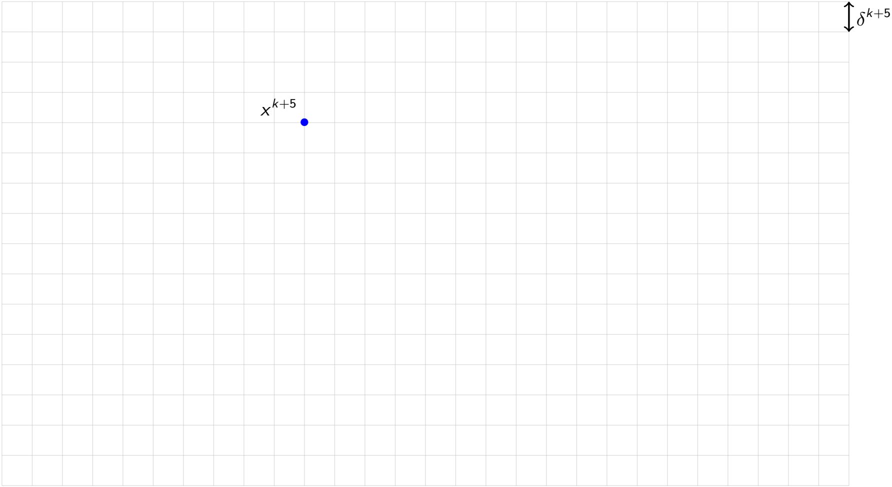
SUCCESS: UPDATE $\delta^{k+4} = \delta^{k+3}$

Coordinate search (CS) for single objective optimization



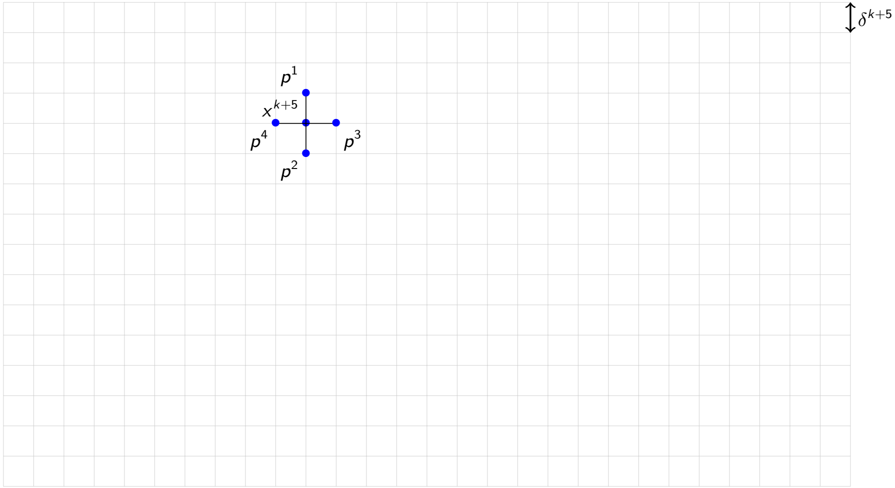
POLL: GENERATION AND EVALUATION

Coordinate search (CS) for single objective optimization



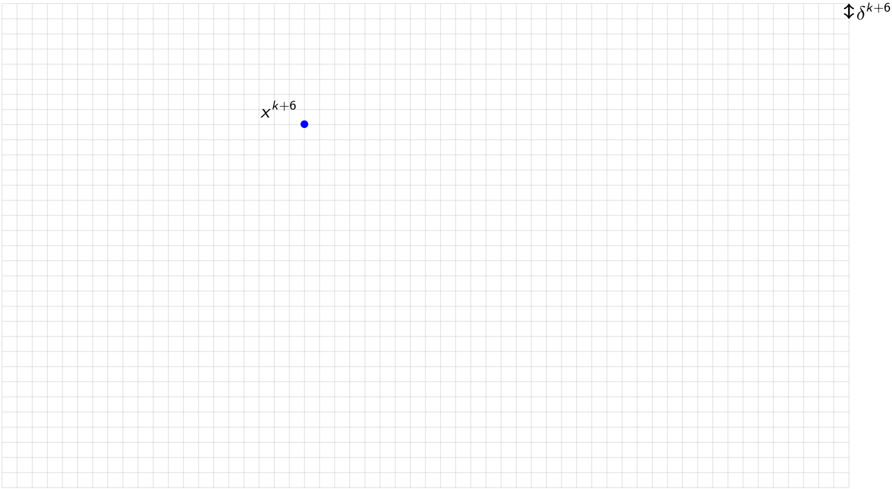
SUCCESS: UPDATE $\delta^{k+5} = \delta^{k+4}$

Coordinate search (CS) for single objective optimization



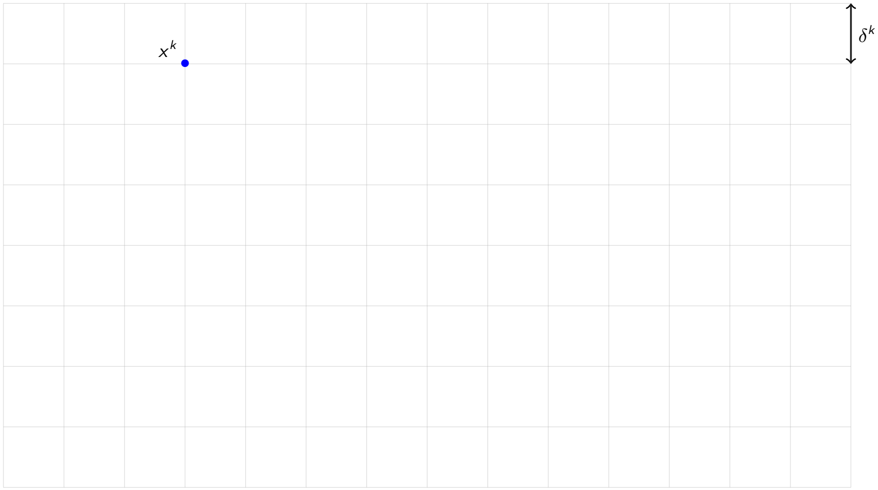
POLL: GENERATION AND EVALUATION

Coordinate search (CS) for single objective optimization



FAILURE: UPDATE $\delta^{k+6} = (1/2)\delta^{k+5}$

Mesh Adaptive Direct Search (MADS) for single objective optimization [Audet and Dennis, 2006]

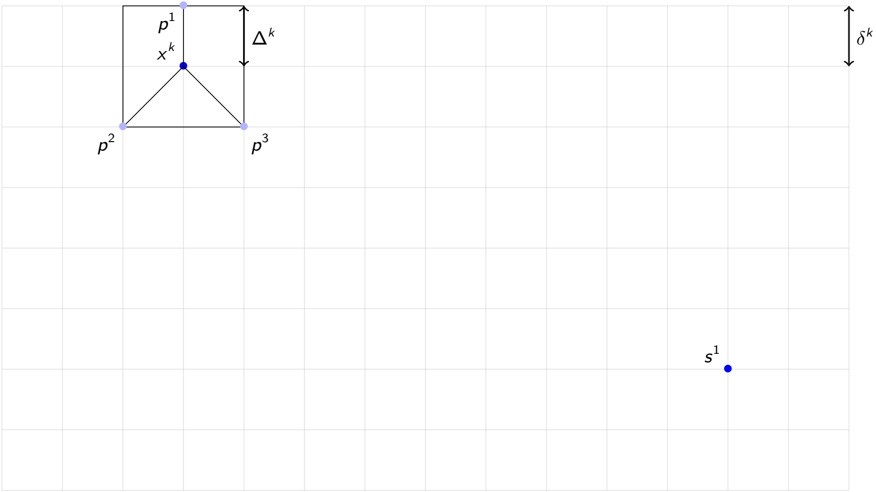


THE MESH OF PARAMETER δ^k

Mesh Adaptive Direct Search (MADS) for single objective optimization [Audet and Dennis, 2006]

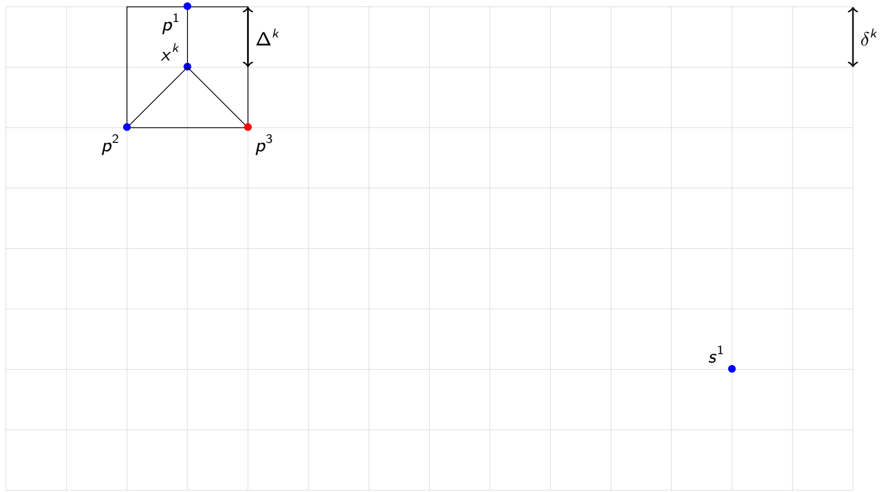
SEARCH

Mesh Adaptive Direct Search (MADS) for single objective optimization [Audet and Dennis, 2006]



POLL: GENERATION ON THE FRAME OF PARAMETER $\Delta^k \geq \delta^k$

Mesh Adaptive Direct Search (MADS) for single objective optimization [Audet and Dennis, 2006]

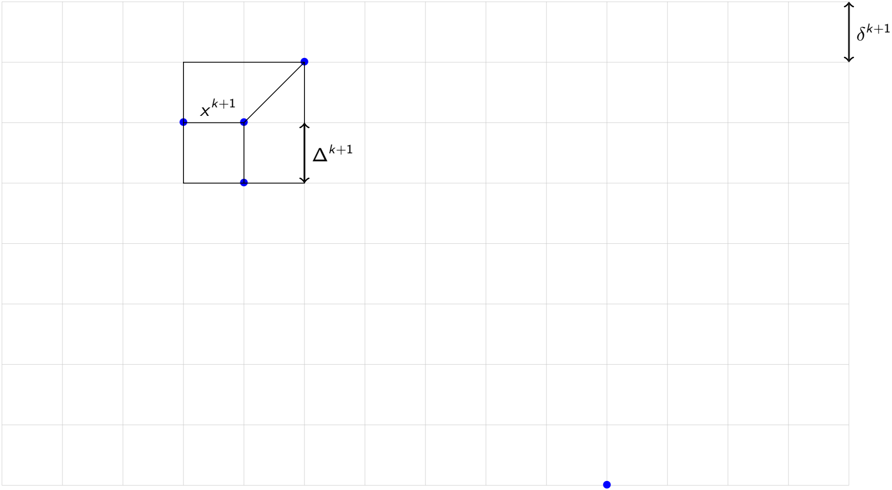


POLL: EVALUATION AND SUCCESS

Mesh Adaptive Direct Search (MADS) for single objective optimization [Audet and Dennis, 2006]

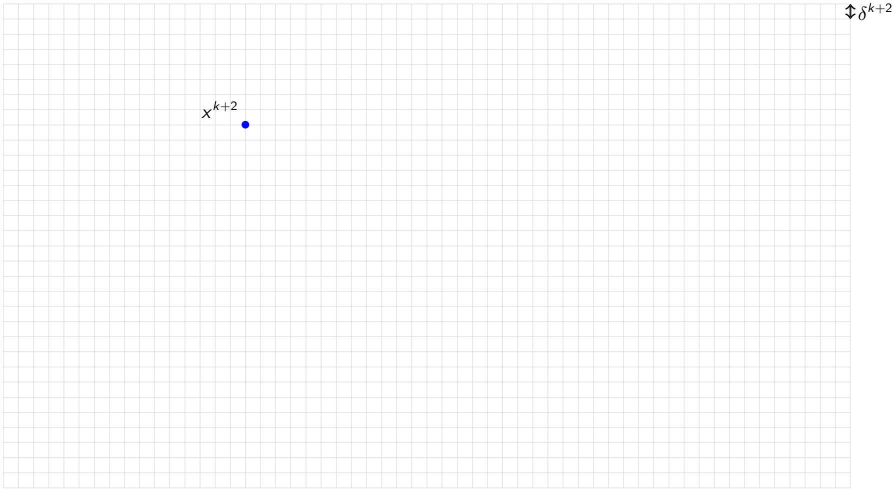
UPDATE: $\Delta^{k+1} \geq \Delta^k$

Mesh Adaptive Direct Search (MADS) for single objective optimization [Audet and Dennis, 2006]



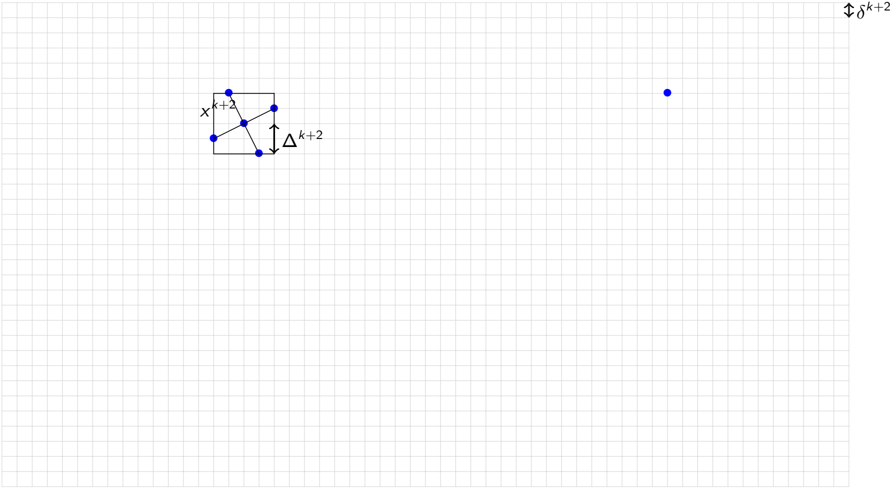
SEARCH AND POLL: FAILURE

Mesh Adaptive Direct Search (MADS) for single objective optimization [Audet and Dennis, 2006]



UPDATE: $\Delta^{k+2} < \Delta^{k+1}$

Mesh Adaptive Direct Search (MADS) for single objective optimization [Audet and Dennis, 2006]



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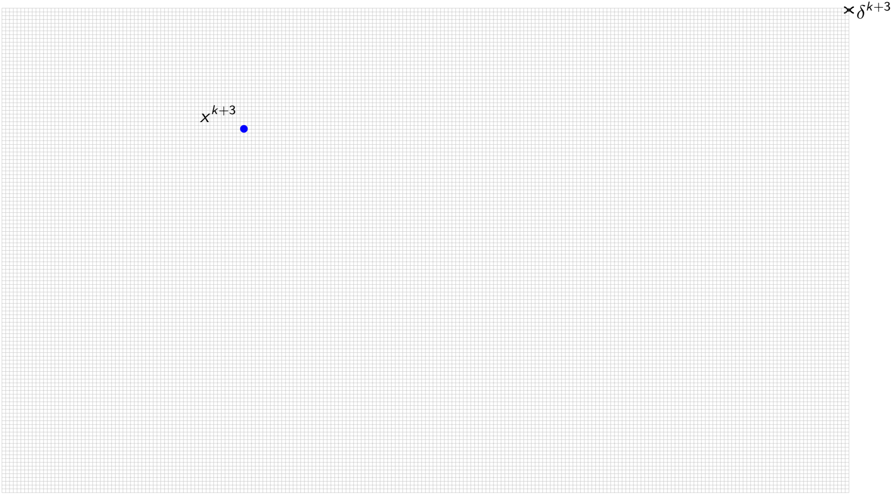

UPDATE: $\Delta^{k+3} < \Delta^{k+2}$

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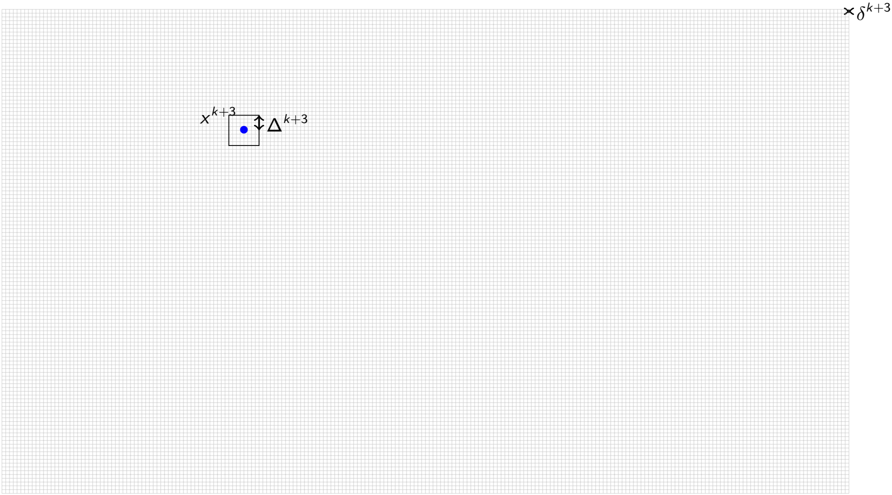

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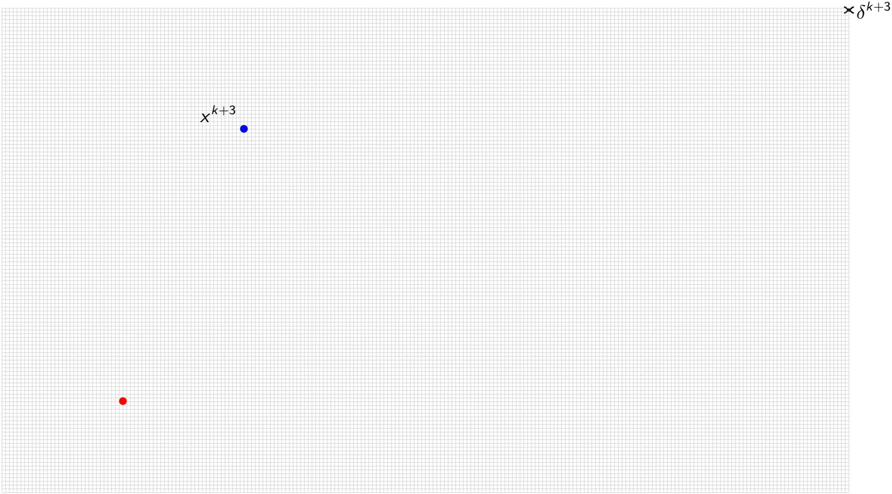

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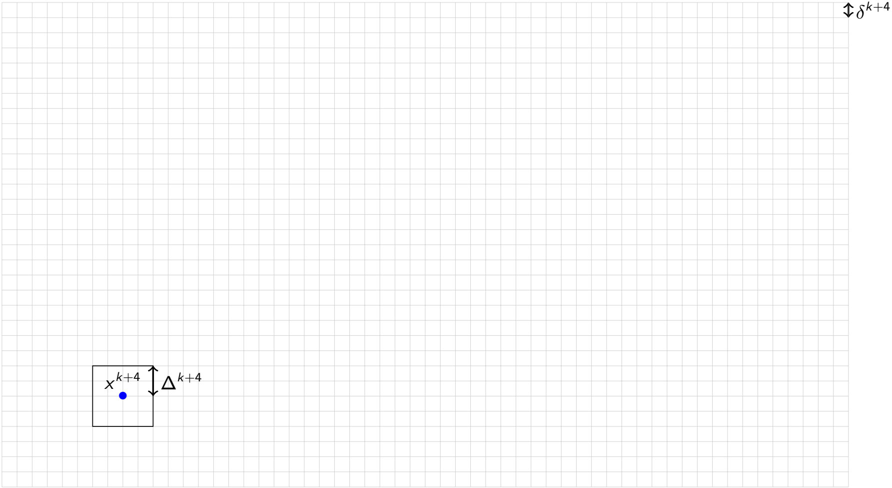

UPDATE: $\Delta^{k+4} \geq \Delta^{k+3}$

Main convergence results for CS and MADS

Theorem (Adapted from [Audet and Hare, 2017])

Let $\Omega = \mathbb{R}^n$. Assume that $f : \mathbb{R}^n \to \mathbb{R}$ has bounded level sets and $f \in \mathcal{C}^1$. Then the sequence of iterates ${x^k}_{k \in \mathbb{R}^n}$ generating by CS will converge to a point $\hat{x}$ satisfying:

$$\nabla f(\hat{x}) = 0.$$

For clarity, we consider that $\Omega = \mathbb{R}^n$.

Theorem ([Audet and Dennis, 2006])

Assume that all iterates lie in a compact set. Then there exists a subsequence of iterates $\{x^k\}_{k \in K}$ generated by MADS converging to a point $\hat{x} \in \Omega$.
Assume that f is locally Lipschitz near $\hat{x} \in \Omega$. Then for all refining directions $d \in \mathbb{R}^n$,

$$f^o(\hat{x}; d) = \lim_{y \to \hat{x}} \sup_{t \searrow 0} \frac{f(y + td) - f(y)}{t} \geq 0.$$

A first approach to extend direct search methods to multiobjective optimization: scalarization-based approaches

Transform the original problem

$$\begin{array}{ll} \text{Minimize} & f(x) = (f_1(x), f_2(x), \ldots, f_m(x))^\top \\ \text{subject to} & \\ & x \in \Omega \subseteq \mathbb{R}^n \end{array}$$

into a succession of **parameterized** single-objective subproblems

$$\begin{array}{ll} \text{Minimize} & \psi_r(x) = \phi_r \circ f(x) \\ \text{subject to} & \\ & x \in \Omega \subseteq \mathbb{R}^n. \end{array}$$

Existing methods

BiMADS [Audet et al., 2008] and MultiMADS [Audet et al., 2010].

A first approach to extend direct search methods to multiobjective optimization: scalarization-based approaches

Example (Weighted sum formulation (very bad))

$$\begin{array}{ll} \text{Minimize} & \sum_{i=1}^{m} w_i f_i(x) \\ \text{subject to} & \\ & x \in \Omega \subseteq \mathbb{R}^n. \end{array}$$

with $w_i \geq 0$ for $i = 1, 2, \ldots, m$ and $\sum_{i=1}^{m} w_i = 1$.

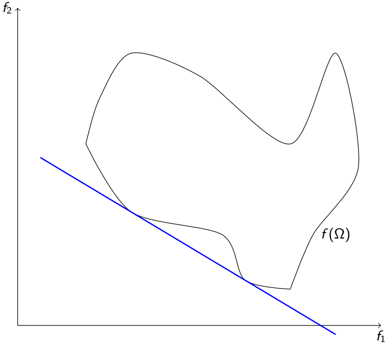

Figure: The weighted sum approach may generate only a subset of the Pareto front.

A first approach to extend direct search methods to multiobjective optimization: scalarization-based approaches

Limitations

- May waste a lot of evaluations to explore a non-interesting part of the objective space.
- Information lost due to the resetting of the algorithm between each subproblem resolution.

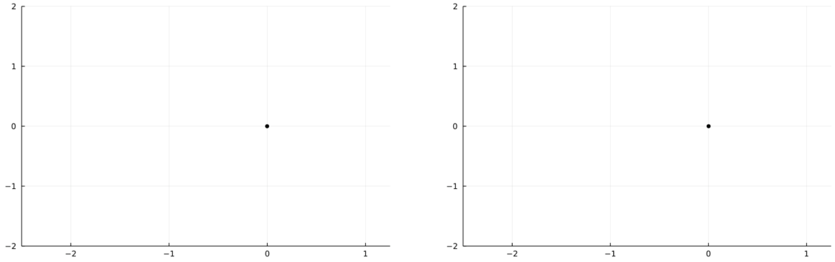

Figure: Deployment of the multiobjective BiMADS (on the left) and DMS (on the right) methods on the Far1 benchmark test function for a maximal budget of 4000 evaluations in the biobjective space.

Second approach: Methods with a posteriori articulation of preferences

Type	Name	Assumptions	Convergence
Direct Search methods	DMS [Custódio et al., 2011] and variants [Dedoncker et al., 2021]	Locally lipschitz	To a point/set
Linesearch approaches	DFMO [Liuzzi et al., 2016a]	Lipschitz continuous	To a set
Implicit filtering	MOIF [Cocchi et al., 2018]	At least $\mathcal{C}^1$	To a point

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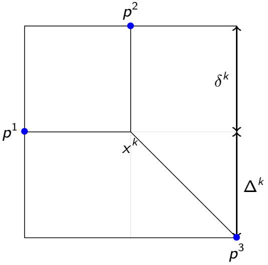
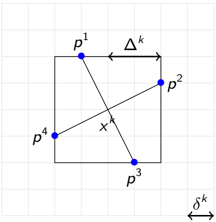
DMulti-MADS [Bigeon et al., 2021]: characteristics

- Is strongly inspired by Direct MultiSearch (DMS) [Custódio et al., 2011] and BiMADS [Audet et al., 2008].
- Does not aggregate any of the objective functions.
- Handles more than 2 objectives, contrary to BiMADS.
- Converges to a set of locally optimal Pareto points, under mild assumptions.
- Is competitive according to other state-of-the-art algorithms (NSGAII [Deb et al., 2000], DMS, MOIF [Cocchi et al., 2018], BiMADS).

DMulti-MADS [Bigeon et al., 2021]: ingredients

- Organized around an (optional) search and a poll.





- Keep a list of non-dominated points (called an iterate list [Custódio et al., 2011])

$$L^k = \{(x^j, \Delta^j), x^j \in \Omega, \Delta^j > 0, j = 1, 2, \ldots, |L^k|\}$$

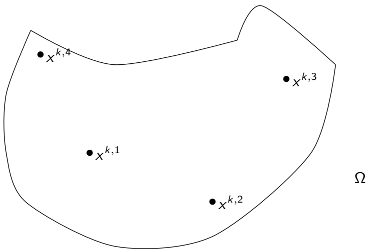
- The selection rule. The poll center (x, Δ) must satisfy

$$\tau^{w^+} \Delta^k_{\max} \leq \Delta \text{ with } \Delta^k_{\max} = \max_{j=1,2,\ldots,|L^k|} \Delta^j$$

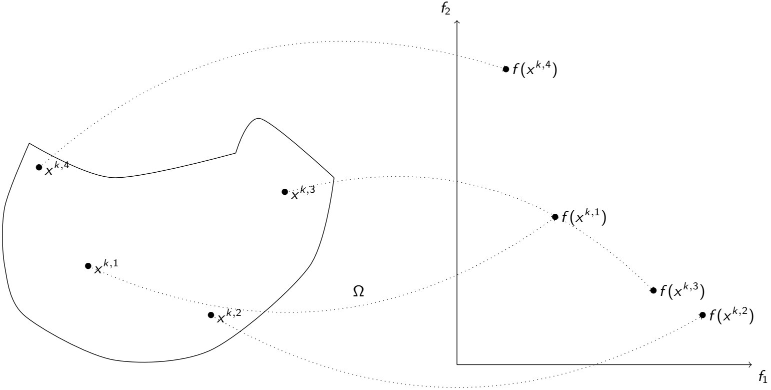
with $\tau \in \mathbb{Q} \cap (0, 1)$ and $w^+ \in \mathbb{N}$ (most of the time, $\tau = \dfrac{1}{2}$).

- Success when $t \prec x^k$.

DMulti-MADS: an illustration

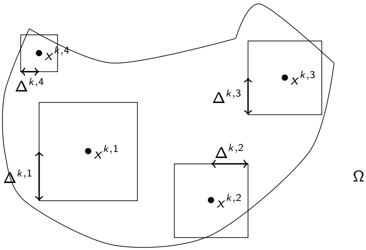


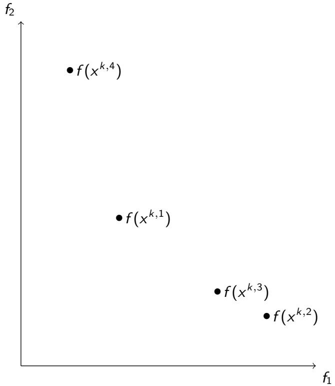
DMulti-MADS: an illustration



DMulti-MADS: an illustration

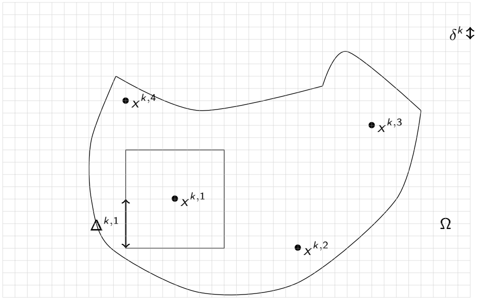
Corresponding frames of parameter $\Delta^{k,j}$

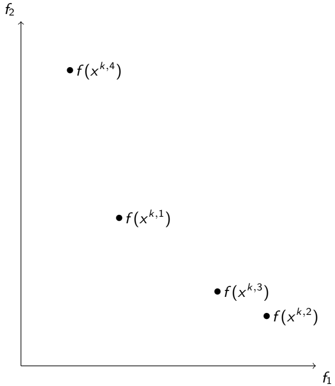




DMulti-MADS: an illustration

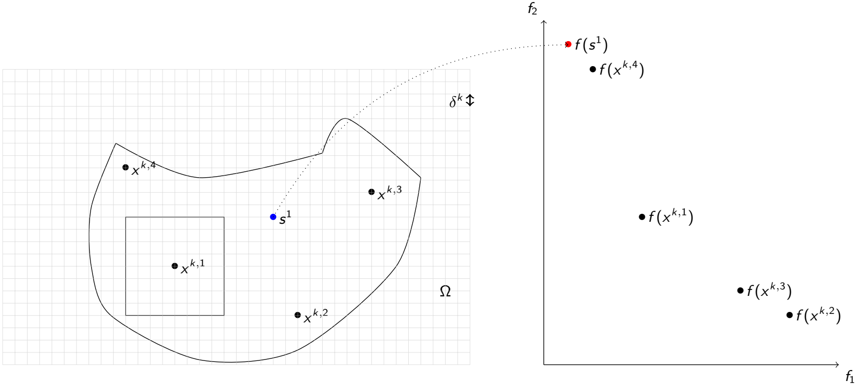
Selection of the current frame center $x^{k,1}$, taking $w^+ = 0$





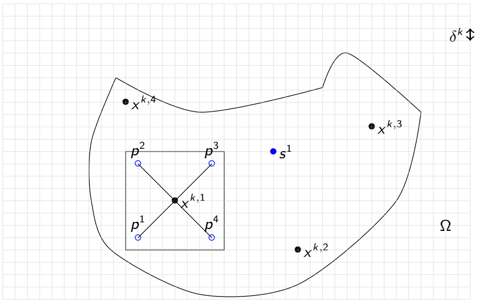
DMulti-MADS: an illustration

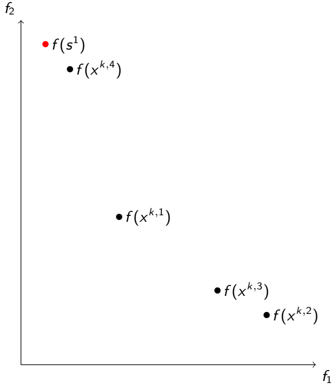
Search step



DMulti-MADS: an illustration

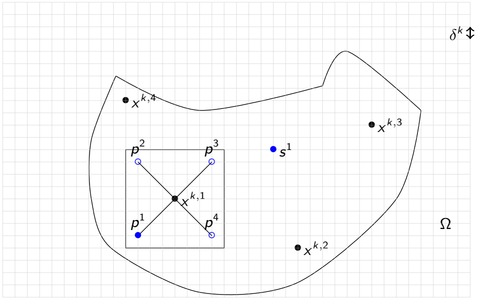
Poll step

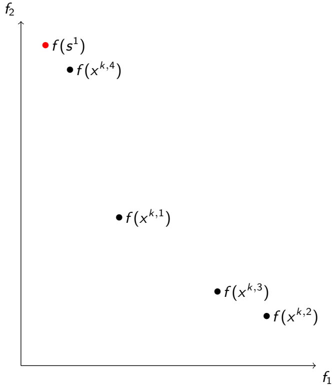




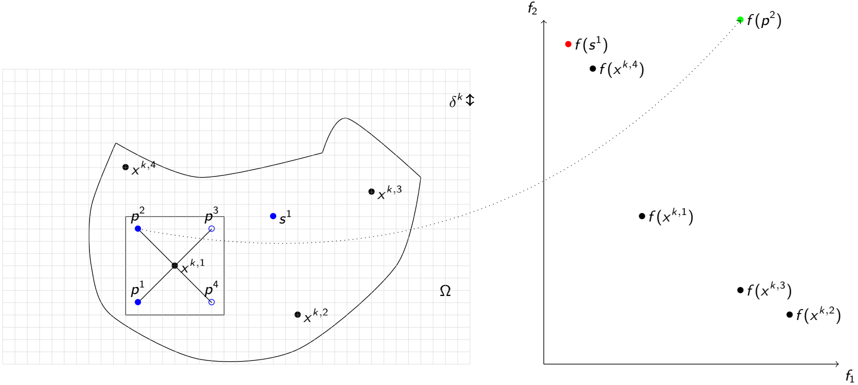
DMulti-MADS: an illustration

Evaluation at p^1 fails !

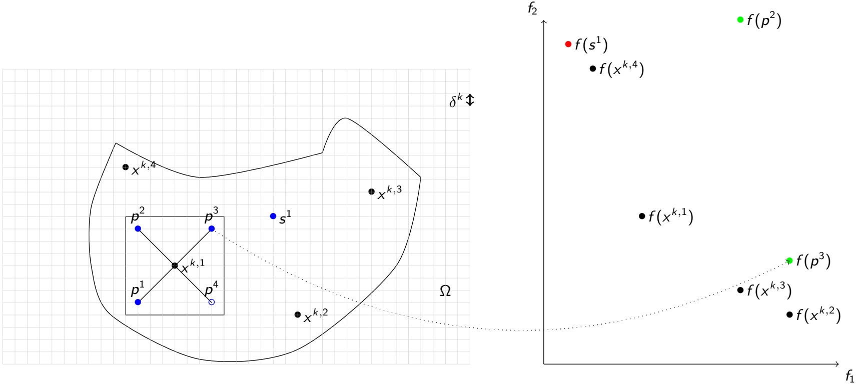




DMulti-MADS: an illustration

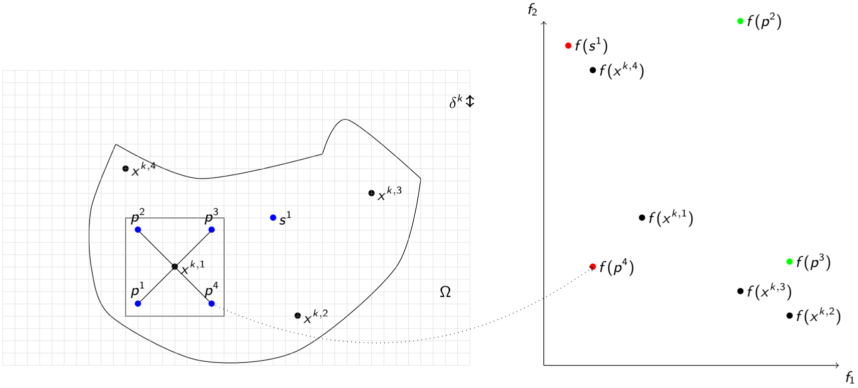


DMulti-MADS: an illustration



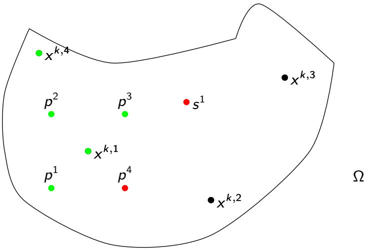
DMulti-MADS: an illustration

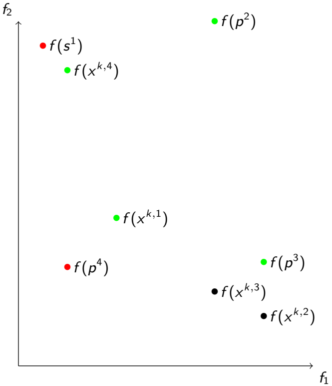
p^4 dominates $x^{k,1}$: success !



DMulti-MADS: an illustration

Keep new non-dominated points: affect them $\Delta \geq \Delta^{k,1}$





DMulti-MADS: Handling constraints [Bigeon et al., 2024]

- Handles relaxable constraints via the use of the constraint violation function [Audet and Dennis, 2009]: defined as

$$h(x) = \begin{cases} \sum_{j \in \mathcal{J}} \max\{c_j(x), 0\}^2 & \text{if } x \in X; \\ +\infty & \text{otherwise.} \end{cases}$$

- Use an adaptive filter-based approach [Bigeon et al., 2024].

Remark

Other direct search algorithms to handle general constraints in multiobjective optimization exist: see for example [Silva and Custódio, 2024]

Main convergence results

For clarity, we consider that $\Omega = \mathbb{R}^n$.

Theorem ([Bigeon et al., 2021])

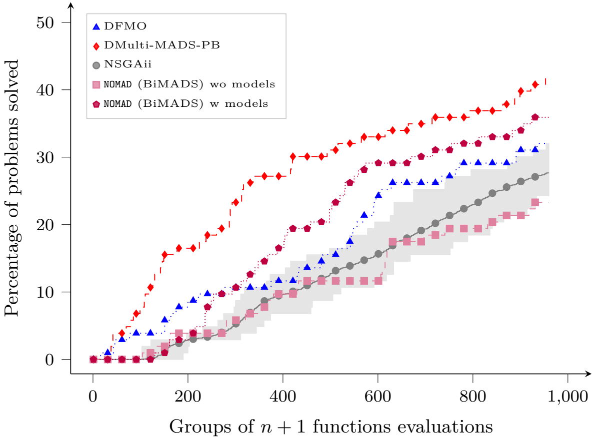
Assume that all iterates lie in a compact set. Then there exists at least a subsequence of iterates $\{x^k\}_{k \in K}$ generated by DMulti-MADS converging to a point $\hat{x} \in \Omega$.
Assume that f is locally Lipschitz near $\hat{x} \in \Omega$. Then for all refining directions $d \in \mathbb{R}^n$, there exists an index $i(d) \in \{1, 2, \ldots, m\}$ such that

$$f_{i(d)}(\hat{x}; d) = \limsup_{\substack{y \to \hat{x} \\ t \searrow 0}} \frac{f_{i(d)}(y + td) - f_{i(d)}(y)}{t} \geq 0.$$

Experiments: Constrained problems

- Use of a constrained benchmark set [Liuzzi et al., 2016b] of functions with $|\mathcal{P}| = 214$ (containing 103 problems with $m = 2$), $n \in [3, 30]$.
- Implementation details: $w^+ = 1$, use of OrthoMads [Abramson et al., 2009] strategy with $n + 1$ directions and granular mesh [Audet et al., 2019].
- Evaluations by hypervolume-based data profiles for multiojective optimization [Bigeon et al., 2021].

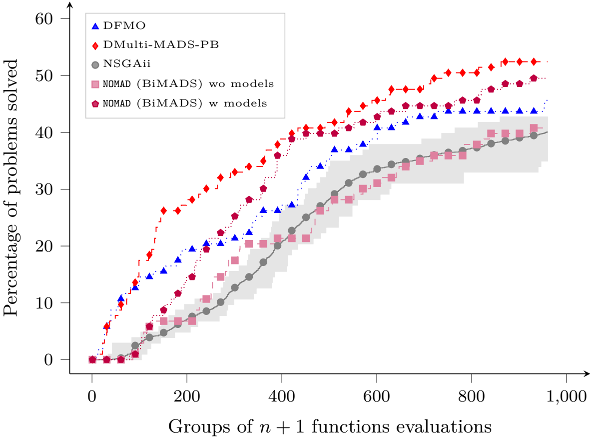
Experiments: comparison of biobjective solvers



(a) $\varepsilon_\tau = 5 \times 10^{-2}$

Figure: Data profiles using NOMAD (BiMADS), DFMO, DMulti-MADS-PB and NSGA-II obtained on 103 biojective analytical problems with 30 different runs of NSGA-II.

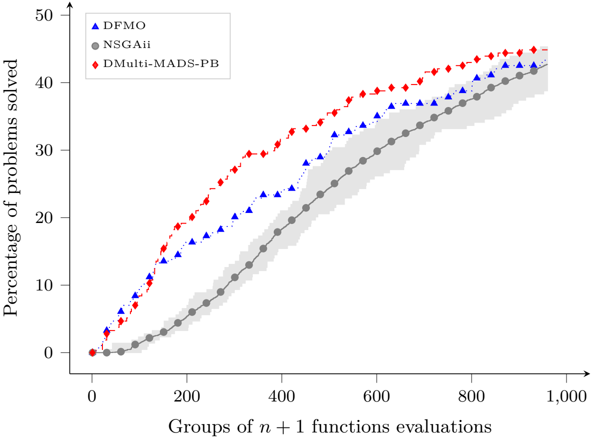
Experiments: comparison of biobjective solvers



(a) $\varepsilon_\tau = 10^{-1}$

Figure: Data profiles using NOMAD (BiMADS), DFMO, DMulti-MADS-PB and NSGA-II obtained on 103 biojective analytical problems with 30 different runs of NSGA-II.

Experiments: comparison of multiobjective solvers



(a) $\varepsilon_\tau = 10^{-1}$

Figure: Data profiles using DFMO, DMulti-MADS-PB and NSGA-II obtained on 214 multiobjective analytical problems with 30 different runs of NSGA-II.

Real world problems: SOLAR8 and SOLAR9 [Lemyre Garneau, 2015]

Characteristics

- Simulate a solar plant.
- SOLAR8 : Maximize absorbed energy and minimize cost; 13 variables (with 2 integers), $m = 2$, $|\mathcal{J}| = 9$.
- SOLAR9 : Maximize power and minimize losses; 29 variables (with 7 integers), $m = 2$, $|\mathcal{J}| = 17$.
- An evaluation $\approx 19s$.

Tests

- Run for a total of 5000 evaluations ($\approx$ 1 day).
- Use normalized hypervolume indicator to see the evolution of the algorithms.
- All algorithms start from an infeasible point.
- Fix integer variables.

Real world problems: SOLAR8 and SOLAR9 [Lemyre Garneau, 2015]

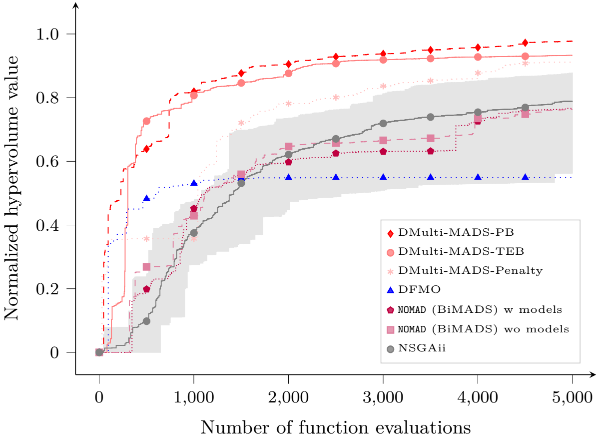


Figure: Convergence profiles for the SOLAR8 problem (fixing integer variables) using DFMO, DMulti-MADS, NOMAD (BiMADS) and NSGA-II with 10 different runs of NSGA-II for a maximal budget of 5, 000 evaluations

Real world problems: SOLAR8 and SOLAR9 [Lemyre Garneau, 2015]

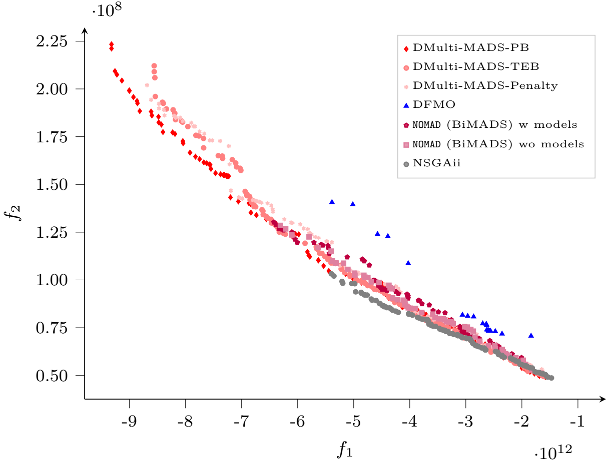


Figure: Pareto front approximations obtained at the end of the resolution of SOLAR8 (fixing integer variables) for DFMO, DMulti-MADS, `NOMAD` (BiMADS) and an instance of NSGA-II in the objective space.

Real world problems: SOLAR8 and SOLAR9 [Lemyre Garneau, 2015]

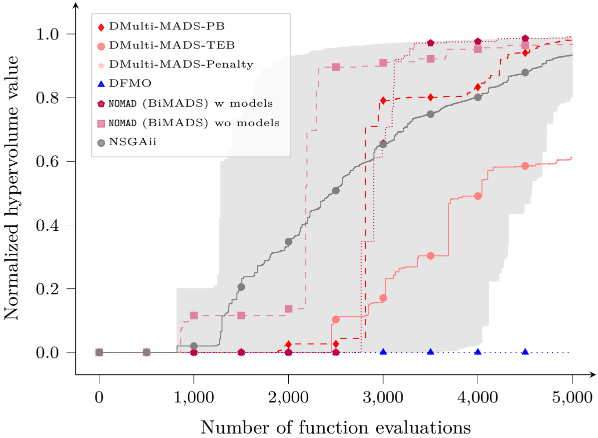


Figure: Convergence profiles for the SOLAR9 problem (fixing integer variables) using DFMO, DMulti-MADS, NOMAD (BiMADS) and NSGA-II with 10 different runs of NSGA-II for a maximal budget of 5, 000 evaluations

Real world problems: SOLAR8 and SOLAR9 [Lemyre Garneau, 2015]

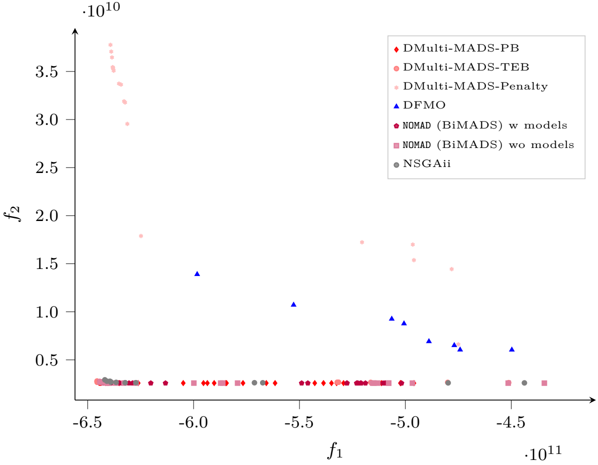


Figure: Pareto front approximations obtained at the end of the resolution of SOLAR9 (fixing integer variables) for DFMO, DMulti-MADS, NOMAD (BiMADS) and an instance of NSGA-II in the objective space.

Extensions

Search strategies for multiobjective direct search methods

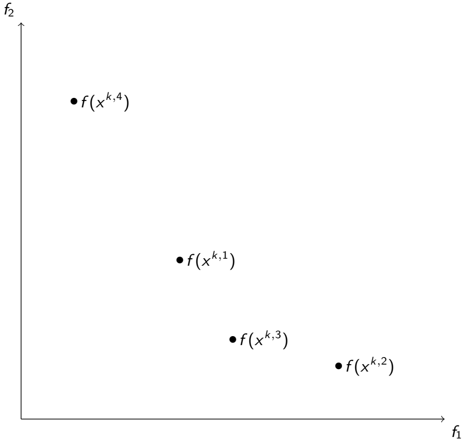
Various search strategies have been implemented in the single-objective case

- Quadratic search [Conn and Le Digabel, 2013, Custódio et al., 2010, Van Dyke and Asaki, 2013].
- Global search strategies [Custódio and Madeira, 2015, Talgorn et al., 2018]
- Nelder-Mead search [Audet and Tribes, 2018].

Idea

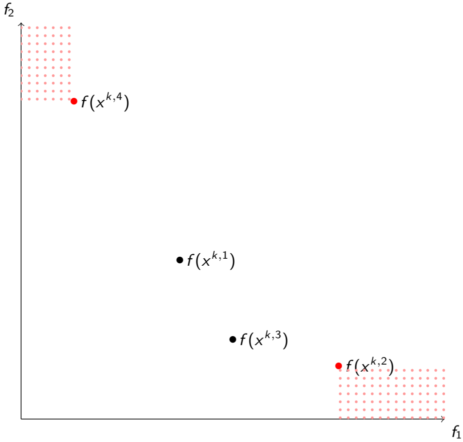
Scalarization-based approaches may be interested, if one manages to use a moderate budget of evaluations.

MultiMADS strategy (inspired by [Audet et al., 2010, Audet et al., 2008])



MultiMADS strategy (inspired by [Audet et al., 2010, Audet et al., 2008])

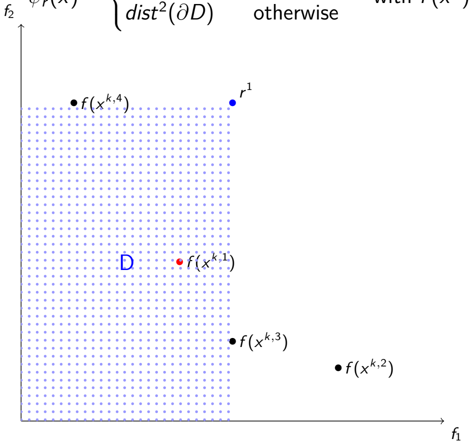
If $x^k \in \arg\min_{x \in F^k} f_{i_0}(x)$, set : $\psi(x) = f_{i_0}(x)$



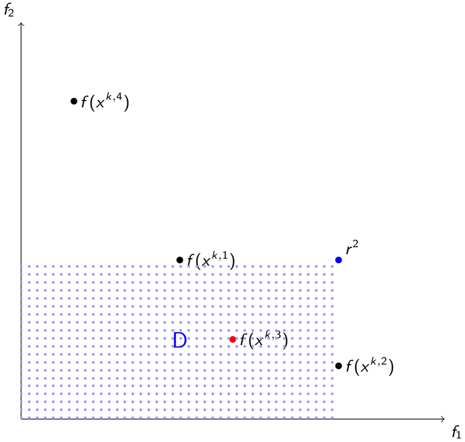
MultiMADS strategy (inspired by [Audet et al., 2010, Audet et al., 2008])

Otherwise, set:

$$\psi_r(x) = \begin{cases} -dist^2(\partial D) & \text{if } f(x) \in D, \\ dist^2(\partial D) & \text{otherwise} \end{cases} \quad \text{with } f(x^k) \prec\prec r.$$



MultiMADS strategy (inspired by [Audet et al., 2010, Audet et al., 2008])



DoM search strategy (inspired by [Li and Yao, 2017])

Illustration of a dominance move

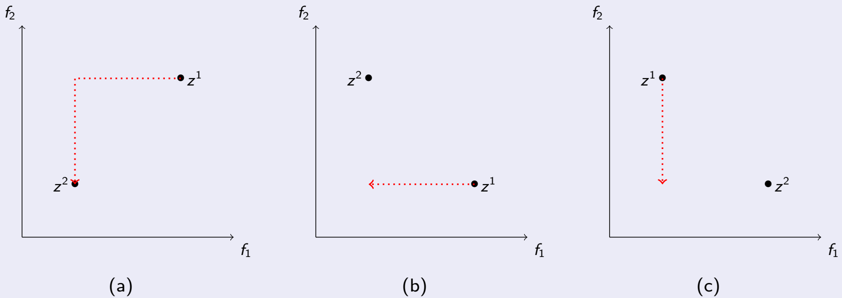
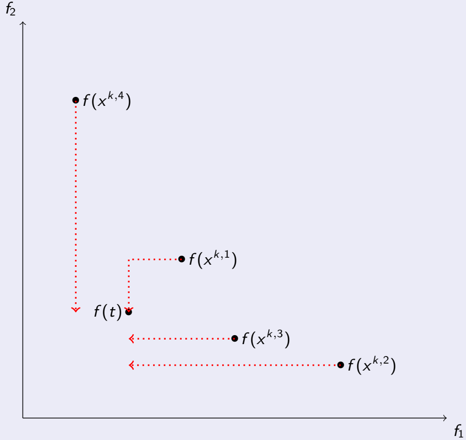


Figure: Representation of a dominance move for objective vector z^1 to dominate objective vector z^2 for a biobjective minimization problem.

DoM search strategy (inspired by [Li and Yao, 2017])

Idea

Maximize the minimum dominance move from each point of the current solution set to a new candidate.



DoM search strategy (inspired by [Li and Yao, 2017])

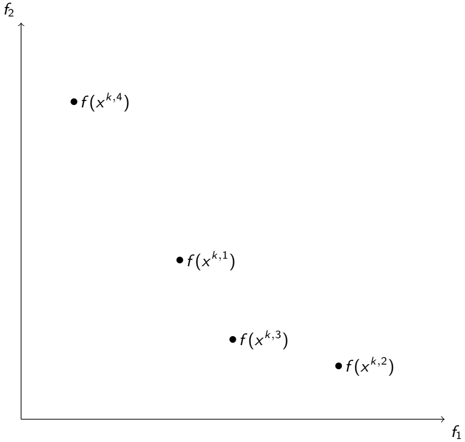
The formula

Set:

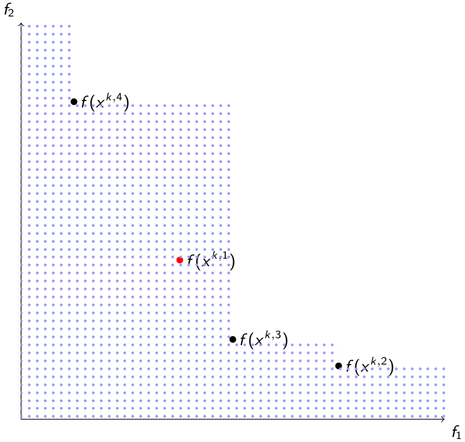
$$\psi_L(x) = \begin{cases} -\min_{y \in L} \sum_{i=1}^{m} \max(0, f_i(y) - f_i(x)) & \text{if there is no } y \in L \text{ such that} \\ & f_i(y) \le f_i(x),\ \ i = 1, 2, \ldots, m \\ \min_{y \in L} \sum_{i=1}^{m} \max(0, f_i(x) - f_i(y)) & \text{otherwise;} \end{cases}$$

with $L \in \left\{ F^k \setminus \{x^k\}, I^k \setminus \{x^k\}, \{x^k\} \right\}$.

DoM search strategy (inspired by [Li and Yao, 2017])



DoM search strategy (inspired by [Li and Yao, 2017])



Which single-objective subsolvers to use ?

Quadratic search strategy

1. Build local quadratic models of the constraints and the scalarization function.
2. Solve a QCQP to obtain a new candidate.

Nelder-Mead search strategy

1. Build an ordered simplex using the scalarization function.
2. Apply a succession of substeps to update the simplex and explore the decision space: reflection - outside contraction - expansion.

Mixed-integer multiobjective optimization: adapting the mesh

The granular mesh [Audet et al., 2019]

- The mesh size parameter δ^k and frame size parameter Δ^k are vectors in $\mathbb{R}^n$ such that:

$$\delta_i^k = 10^{b_i^k - |b_i^0 - b_i^k|} \text{ and } \Delta_i^k = a_i^k \times 10^{b_i^k}, \forall i = 1, 2, \ldots, n$$

  with $a_i^k \in \{1, 2, 5\}$ and $b_i^k \in \mathbb{Z}$.
- Use the `decrease` and `increase` functions defined by:

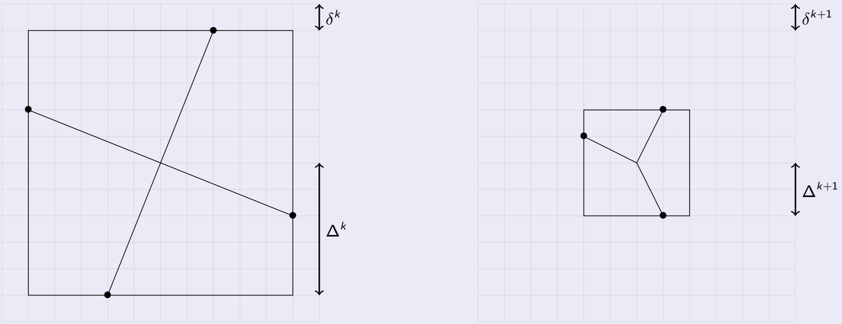
$$\texttt{decrease}(a \times 10^b) = \begin{cases} 5 \times 10^{b-1} & \text{if } a = 1, \\ 1 \times 10^b & \text{if } a = 2, \\ 2 \times 10^b & \text{if } a = 5, \end{cases}$$

  and

$$\texttt{increase}(a \times 10^b) = \begin{cases} 2 \times 10^b & \text{if } a = 1, \\ 5 \times 10^b & \text{if } a = 2, \\ 1 \times 10^{b+1} & \text{if } a = 5. \end{cases}$$

Mixed-integer multiobjective optimization: adapting the mesh

Successive decreases of the mesh (adapted from [Audet et al., 2019])



Mixed-integer multiobjective optimization: adapting the mesh

Update the mesh

- In case of failure (adapted from [Audet et al., 2019]),

$$\Delta_i^{k+1} = \begin{cases} \texttt{decrease}(\Delta_i^k) & \text{if } i \in I^c, \\ \max\left(1, \texttt{decrease}(\Delta_i^k)\right) & \text{if } i \in I^z. \end{cases}$$

- Increasing the mesh is slightly more complicated and uses a combination of `increase` and previous success direction [Audet et al., 2019]:

$$\Delta_i^{k+1} = \begin{cases} \texttt{increase}(\Delta_i^k) & \text{under some conditions,} \\ \Delta_i^k & \text{otherwise.} \end{cases}$$

- $a^0 \in \{1, 2, 5\}^n$ and $b^0 \in \mathbb{Z}^n$ are initialized using $l \in (\mathbb{R} \cup \{-\infty\})^n$ and $u \in (\mathbb{R} \cup \{+\infty\})^n$.
- The mesh size parameter $\delta^k \in \mathbb{R}^n$ is updated using the following formula (adapted from [Audet et al., 2019]):

$$\delta_i^{k+1} = \begin{cases} 10^{b_i^{k+1} - |b_i^0 - b_i^{k+1}|} & \text{if } i \in I^c, \\ \max\left(1, 10^{b_i^{k+1} - |b_i^0 - b_i^{k+1}|}\right) & \text{if } i \in I^z. \end{cases}$$

Mixed-integer strategy - Convergence profiles: Solar 9

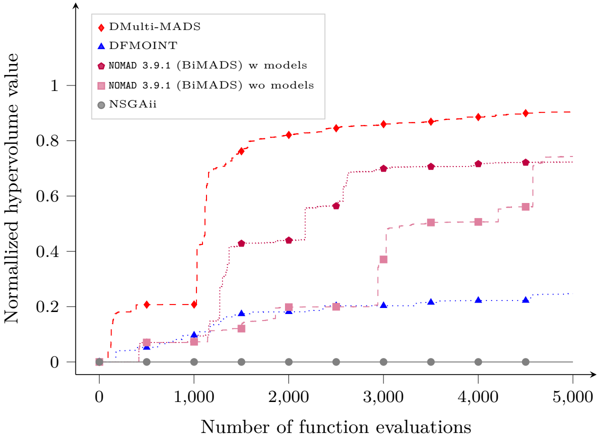


Figure: Convergence profiles for the SOLAR9 problem using DFMOINT, DMulti-MADS, NOMAD (BiMADS) and NSGA-II with 10 different runs of NSGA-II for a maximal budget of 5,000 evaluations.

Mixed-integer strategy - Pareto fronts plot: Solar 9

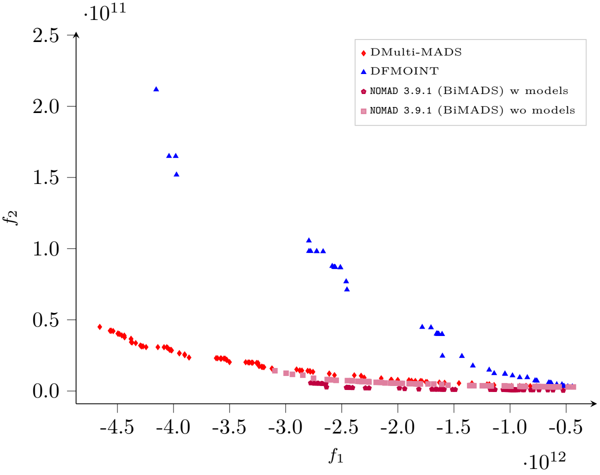


Figure: Pareto front approximations obtained at the end of the resolution of SOLAR9 for DFMOINT, DMulti-MADS, `NOMAD` (BiMADS) and an instance of NSGA-II in the objective space.

Impact of a Nelder-Mead search strategy: Solar 8

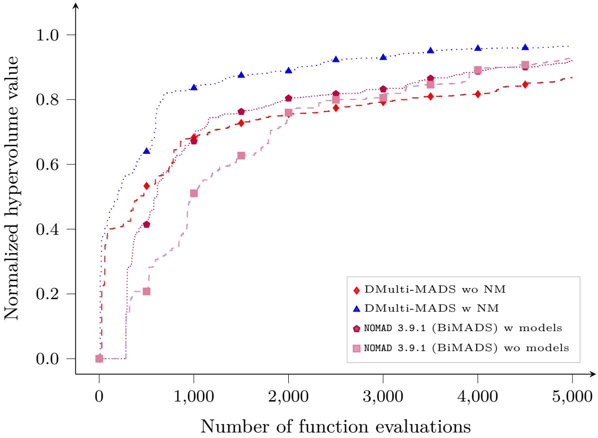


Figure: Convergence profiles obtained for SOLAR8 for DMulti-MADS with and without Nelder-Mead search and `Nomad 3.9.1` (BiMADS).

Impact of a Nelder-Mead search strategy: Solar 8

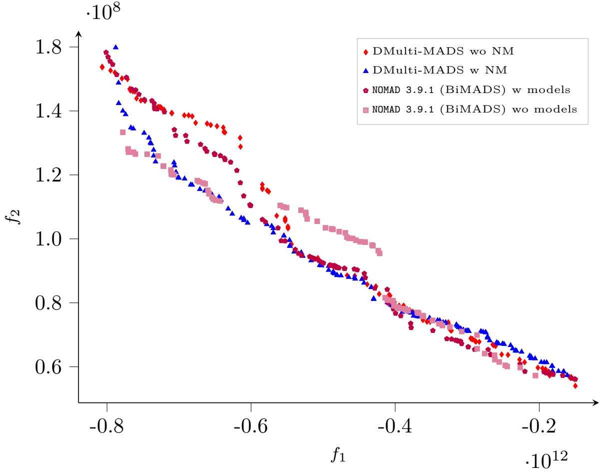


Figure: Pareto fronts obtained for SOLAR8 for DMulti-MADS with and without Nelder-Mead search and `Nomad 3.9.1` (BiMADS).

Conclusion

- Direct search algorithms are a class of flexible and robust methods to solve blackbox multiobjective optimization problems.
- They represent an interesting alternative to classical heuristics (evolutionary algorithms particule-swarm).
- The `Nomad` software proposes a state-of-the-art implementation of the MADS and DMulti-MADS algorithm: see `https://www.gerad.ca/fr/software/nomad/`.
- If you have some information on the structure of your problem, use it !

Future research perspectives

- Solve larger problems by using parallelism / random subspace projection.
- Tackle stochastic objectives and/or constraints.

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