Service facility location optimization during a pandemic

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GERAD Seminar

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About

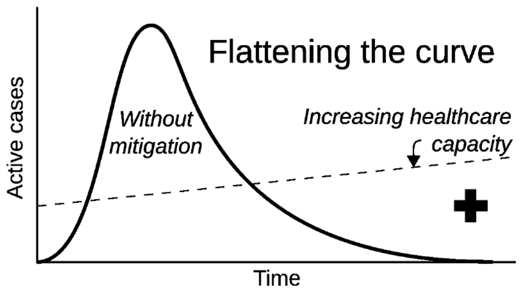
- *Transportation science* and *location science*,
- Design and management of *large-scale networks*,
- Disruptive technologies in transportation.
  - Refueling\charging stations, WIM systems, telecom networks, refugee camp network
  - Attended home delivery, public transportation of goods

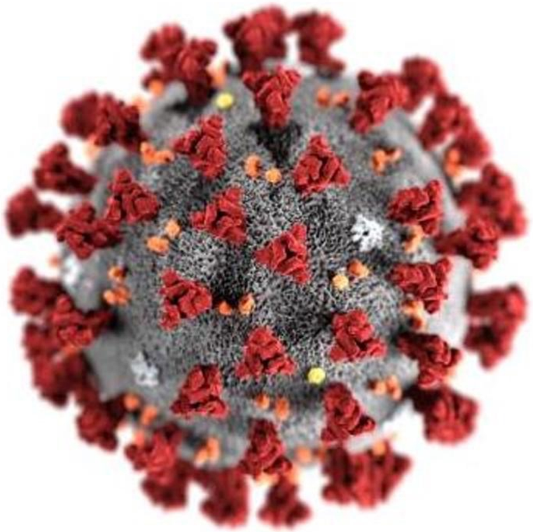
- Combinatorial optimization, polyhedral analysis, decomposition techniques, Benders decomposition, column generation, branch-and-cut, branch-and-price and Lagrangian relaxation.

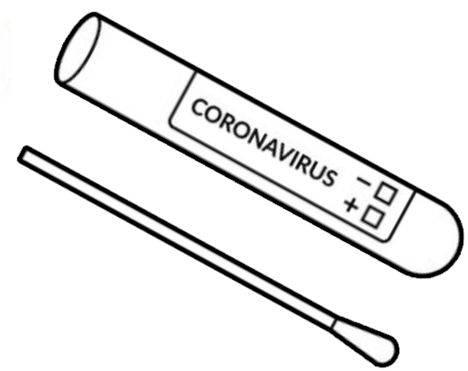
Disruptions and transportation

- Disruptions and transportation
  - Disruptive technologies in transportation
  - Disruptions in transportation

Service facility location optimization during a pandemic



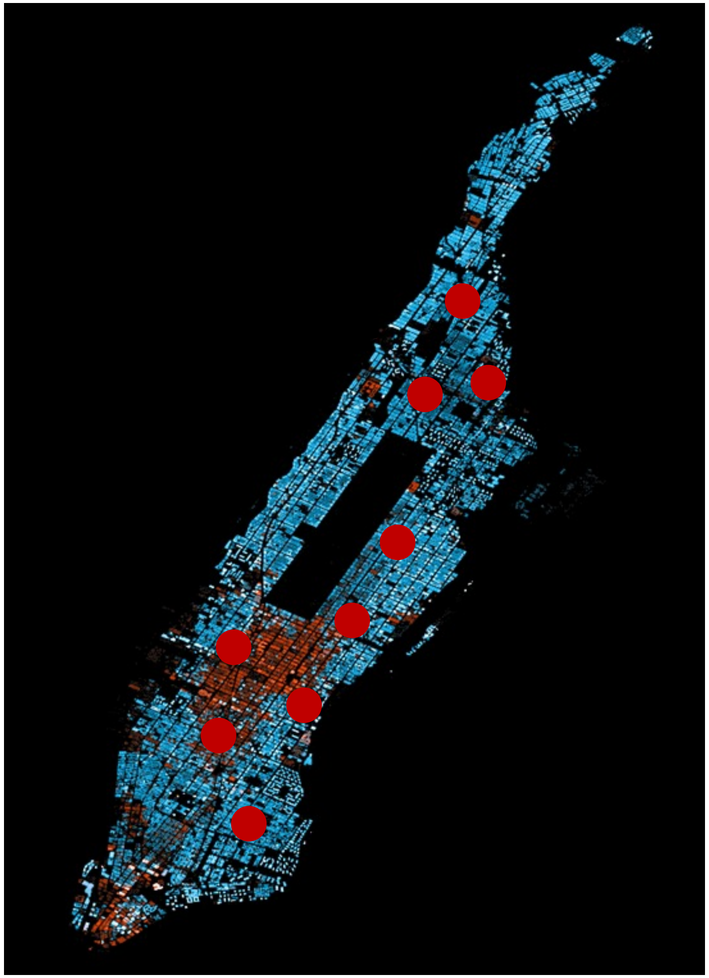




Sources: en.wikipedia.org/wiki/Flattening_the_curve, flylax.com/travelsafely, scwtoday.com/675/features/scwest-pandemic-edition/

Service facility location optimization during a pandemic

- Service can be provided
  - at a facility
  - at a customer's location.



Sources: darkhorseanalytics.com/blog/breathing-city/

Service facility location optimization during a pandemic

- Service can be provided
  - at a facility or
  - at a customer's location.

- The demand can be
  - stationary
  - or mobile.

Sources: darkhorseanalytics.com/blog/breathing-city/

Outline

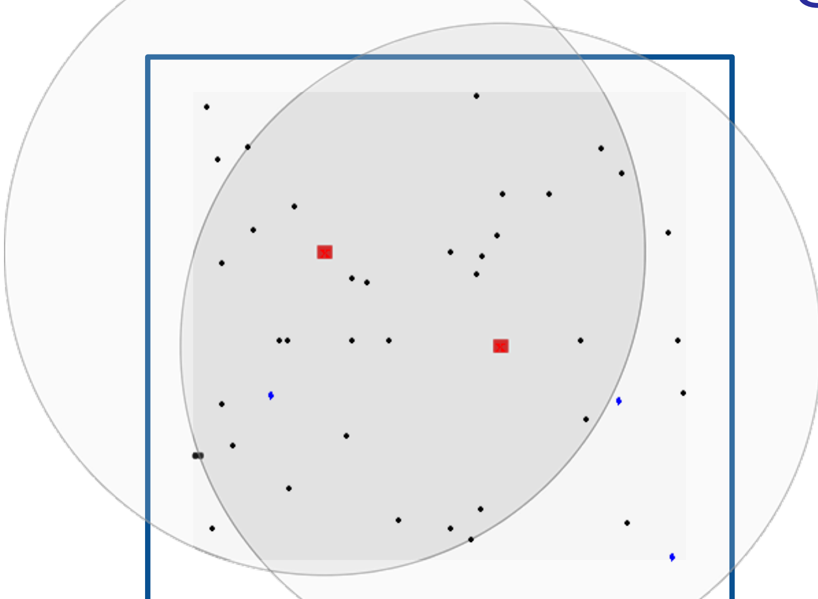
- Service can be provided → • Location-or-routing problem
  - at a facility or
  - at a customer's location.

- Location-or-routing problem
  - Arslan, O. (2021). The location-or-routing problem. *Transportation Research Part B: Methodological*, *147*, 1-21.
  - Branch-and-price algorithm.

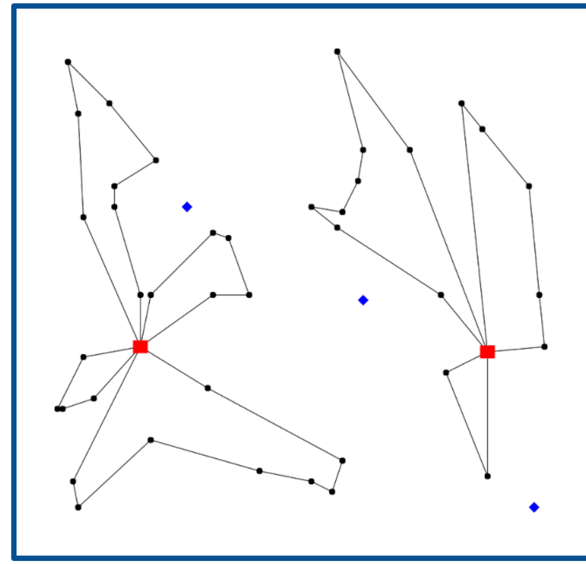
- The demand can be → • Maximum availability service facility location problem
  - stationary
  - or mobile.

- Maximum availability service facility location problem
  - Ali Muffak, MSc student.
  - CIRRELT report.
  - Benders decomposition algorithm.

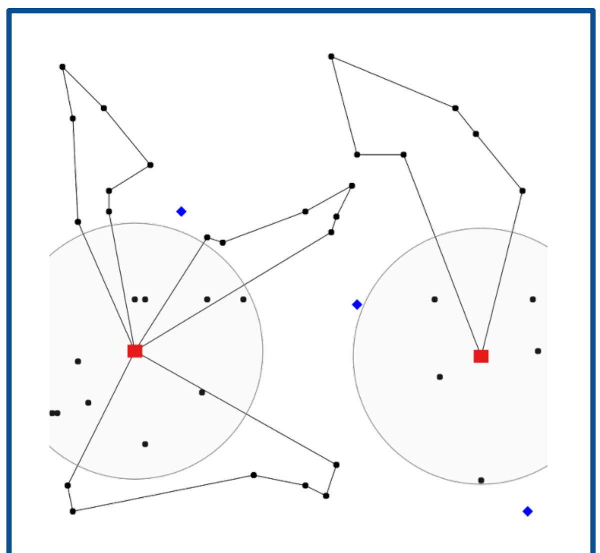
The location-or-routing problem



Location problems (covering, median)



Routing, location-and-routing problem



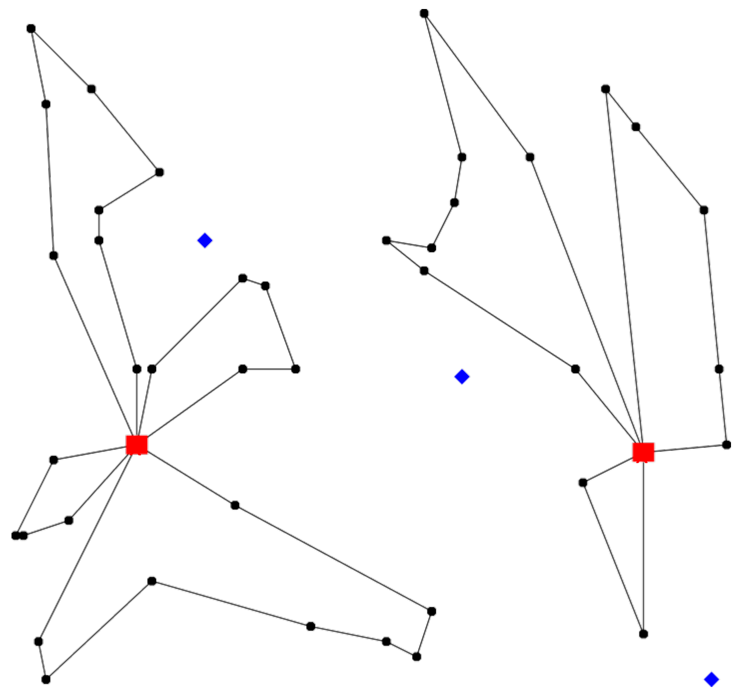
Location-or-routing problem

The location-or-routing problem

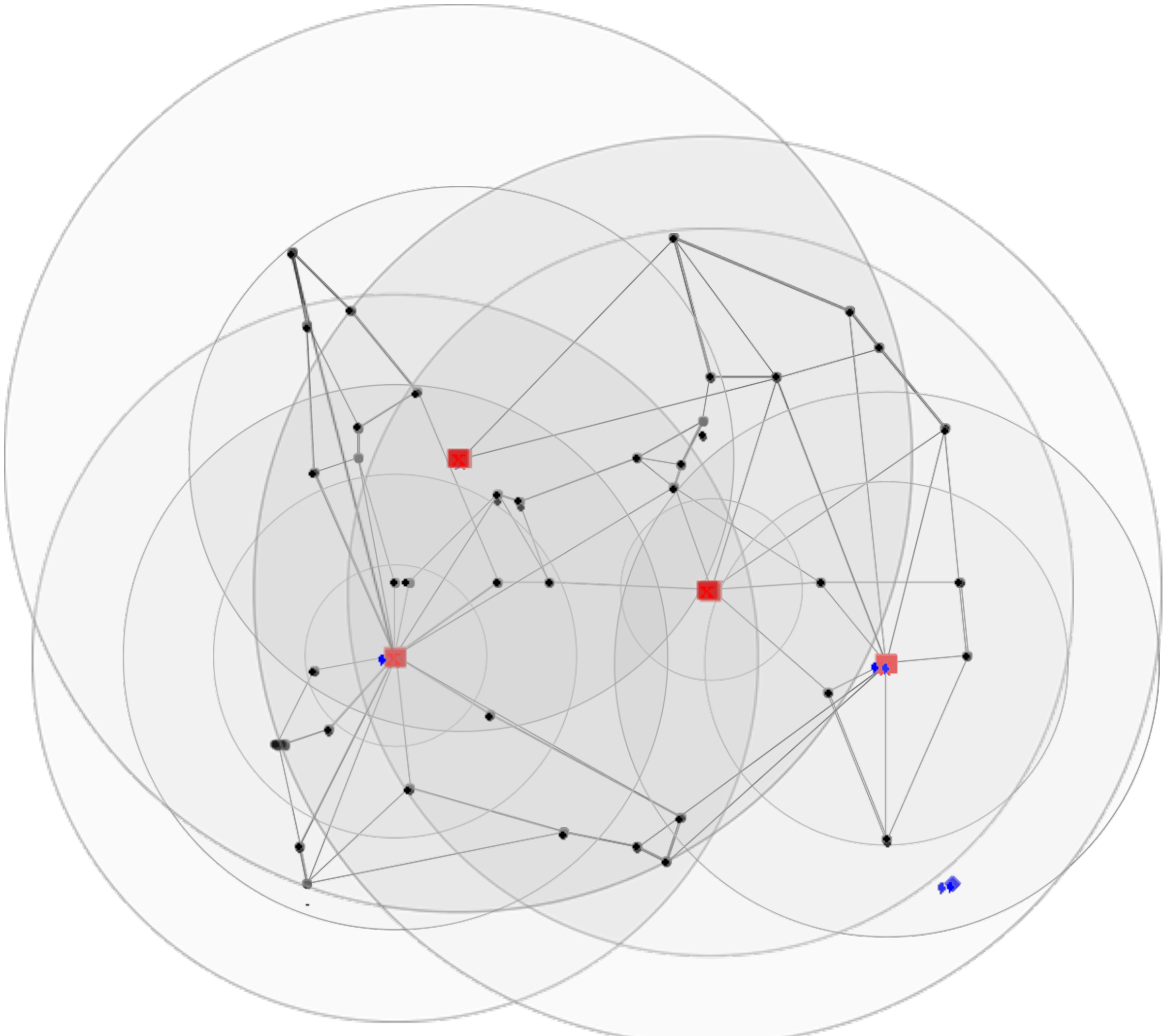
- Medical testing center location and vehicle routing
- Shopping mall/retail store location and shuttle routing
- School location and bus routing
- Urban delivery center location with drone operations

Sources: montreal.ctvnews.ca/mobile/here-s-where-montreal-s-mobile-covid-19-testing-sites-will-be-parked-for-their-last-3-days-1.4987934
www.victoriabuzz.com/2016/10/new-tsawwassen-mills-shopping-mall-eyes-vancouver-island-customers-opens-wedneday/
https://commons.wikimedia.org/wiki/File:School_bus_Montreal_2011.jpg

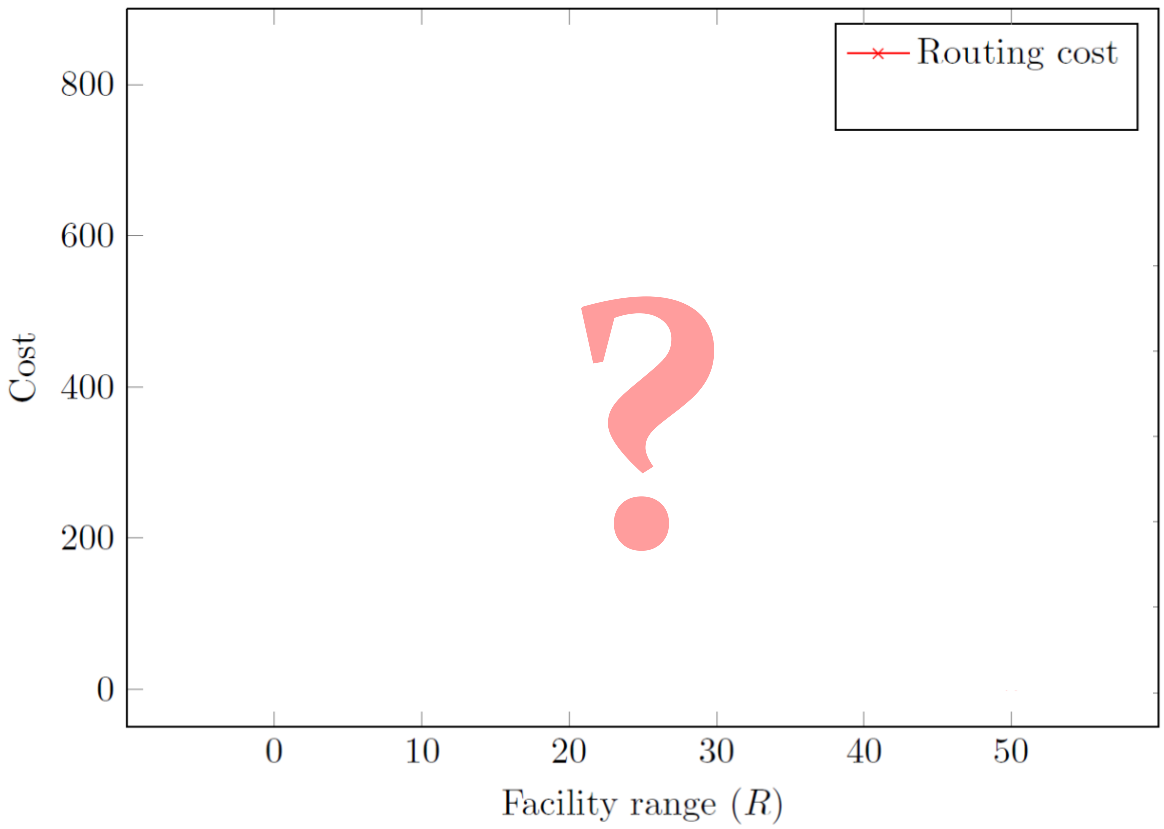
The location-or-routing problem



The location-or-routing problem



The location-or-routing problem

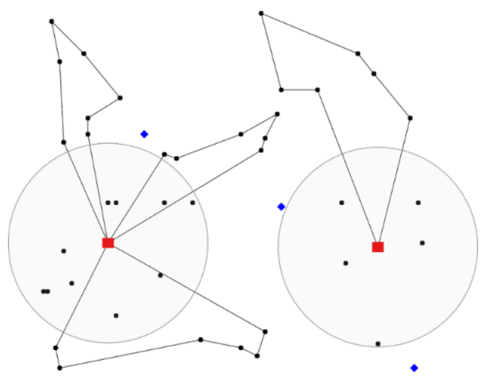


How does the cost change in range?

The location-or-routing problem

Problem definition

- Location-or-routing problem (LoRP) is defined as
  - selecting a set of facilities to open and
  - determining a set of vehicle routes,
    - start at an open facility,
    - visit a subset of customers and
    - return to the same facility,
    - respecting the vehicle capacity and
    - maximum length constraints,



  - such that every customer is covered either by a facility or by a vehicle route and
  - the total cost of opening facilities, routing vehicles and covering the customers by facilities is minimized.

The location-or-routing problem

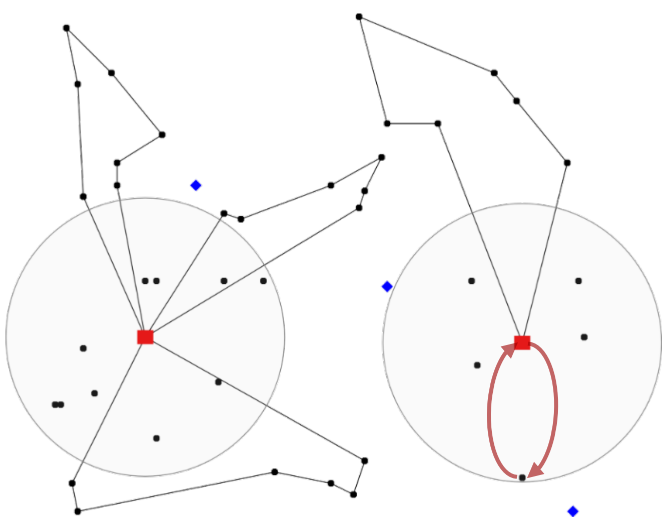
A set covering formulation

$$
\begin{aligned}
\text{minimize} \quad & w_F \sum_{i \in I} f_i x_i + w_R \sum_{p \in \mathcal{P}} d_p y_p + w_A \sum_{j \in J} \sum_{i \in I_j} d_{ij} z_{ij} & \\
\text{subject to} \quad & \sum_{i \in I_j} z_{ij} + \sum_{p \in \mathcal{P}_j} y_p = 1 & j \in J \\
& \sum_{j \in J_i} q_j z_{ij} + \sum_{p \in \mathcal{P}_i} q_p y_p \leq C_i x_i & i \in I \\
& z_{ij} \leq x_i & i \in I, j \in J_i \\
& x_i, y_p, z_{ij} \geq 0 \text{ and integer} & i \in I, j \in J_i, p \in \mathcal{P},
\end{aligned}
$$

The location-or-routing problem

Special cases

- If the coverage range $r_i = 0$ for all $i \in I$, LoRP then transforms into an LRP.
- If the maximum route length of vehicles $T = 0$ and there is no cost for covering customers by facilities, LoRP then transforms into a set covering problem.
- LoRP is a special case of mixed LRP (heterogeneous LRP).



The location-or-routing problem

Location-or-routing as a problem class

$$
\begin{aligned}
\text{minimize} \quad & \underline{w_F} \sum_{i \in I} f_i x_i + \underline{w_R} \sum_{p \in \mathcal{P}} d_p y_p + \underline{w_A} \sum_{j \in J} \sum_{i \in I_j} d_{ij} z_{ij} \\
\text{subject to} \quad & \sum_{i \in I_j} z_{ij} + \sum_{p \in \mathcal{P}_j} y_p = 1 && j \in J \\
& \sum_{j \in J_i} q_j z_{ij} + \sum_{p \in \mathcal{P}_i} q_p y_p \leq \underline{C_i x_i} && i \in I \\
& z_{ij} \leq x_i && i \in I, j \in J_i \\
& x_i, y_p, z_{ij} \geq 0 \text{ and integer} && i \in I, j \in J_i, p \in \mathcal{P},
\end{aligned}
$$

- Uncapacitated location-or-routing problem (ULoRP)
- Set covering location-or-routing problem (SCLoRP)
- Hard-cost minimizing location-or-routing problem (HMLoRP)
- P-median minimizing location-or-routing problem (pMLoRP)
- P-center minimizing location-or-routing problem (pCLoRP)
- Maximum covering location-or-routing problem (MCLoRP)

The location-or-routing problem

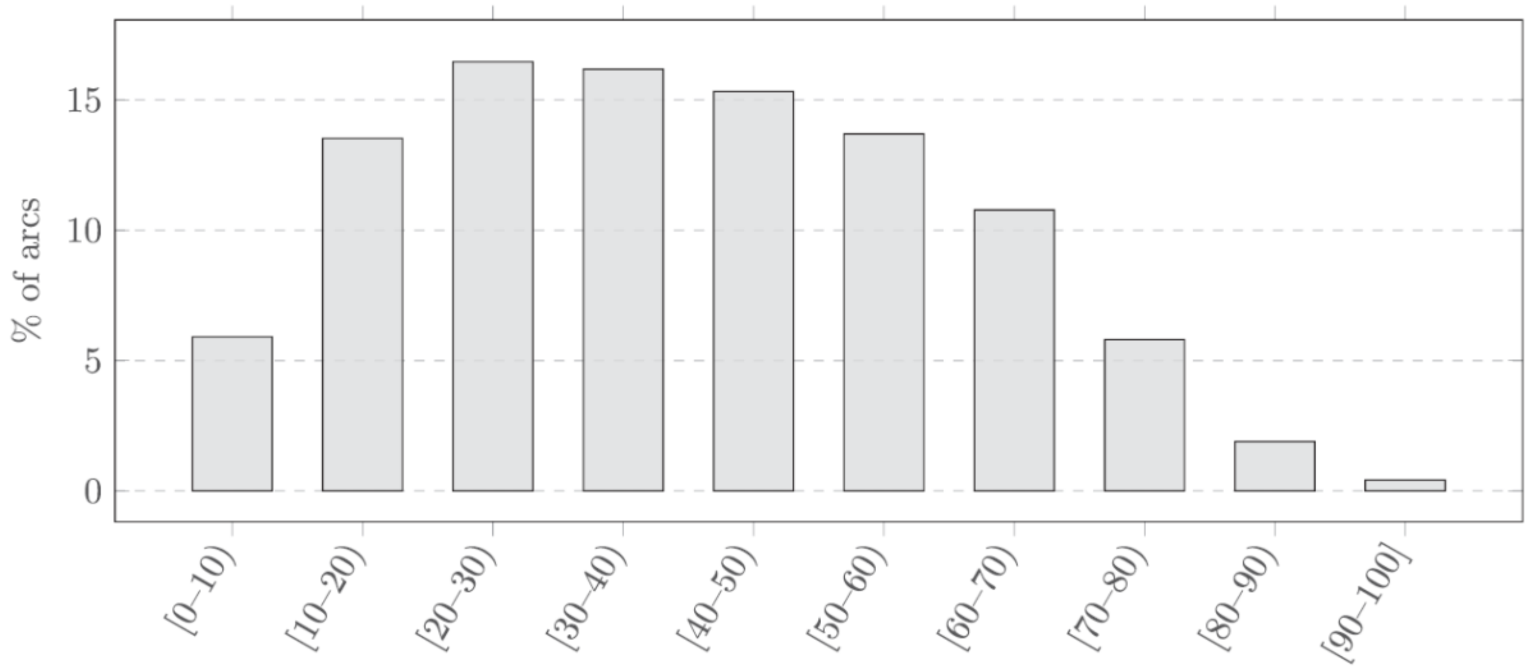
Solution method

- Branch-and-price algorithm.
- The pricing problem is a resource constrained shortest path problem (with two resources, time and load).
- We adopt the pulse algorithm developed by Lozano et al. (2016) for solving the pricing problem.
- We implement a four-stage hierarchical branching.
  - x_i location variables
  - v_i artificial variables (# vehicles)
  - r_{ij}^k implicit flow variables
  - z_{ij} assignment variables
- Upper bound heuristic.

The location-or-routing problem

Computational results

- We use the networks of Akca et al. (2009) and Prodhon (2006).



The location-or-routing problem

Computational results

- We solve HMLoRP and pMLoRP (p = 1, ..., max # facilities).

Dataset	# nodes	# instances	Solution status (% of instances)			
			Optimal	Infeasible	Feasible	Unknown
Prodhon-20-5	20	864	75.7%	24.3%	0.0%	0.0%
Akca-30-5	30	1296	80.8%	19.0%	0.2%	0.0%
Akca-40-5	40	1296	71.7%	26.9%	1.5%	0.0%
Prodhon-50-5	50	1728	46.3%	27.8%	19.6%	6.4%
Prodhon-100-5	100	1296	29.6%	34.3%	12.0%	24.1%
Prodhon-100-10	100	2376	44.3%	10.6%	19.7%	25.5%
Prodhon-200-10	200	2376	33.7%	11.1%	7.1%	48.1%
		11,232	50.4	20.0	10.3	19.3

The location-or-routing problem

Computational results

- Impacts of facility coverage (R) and max.veh.route dist. (T).

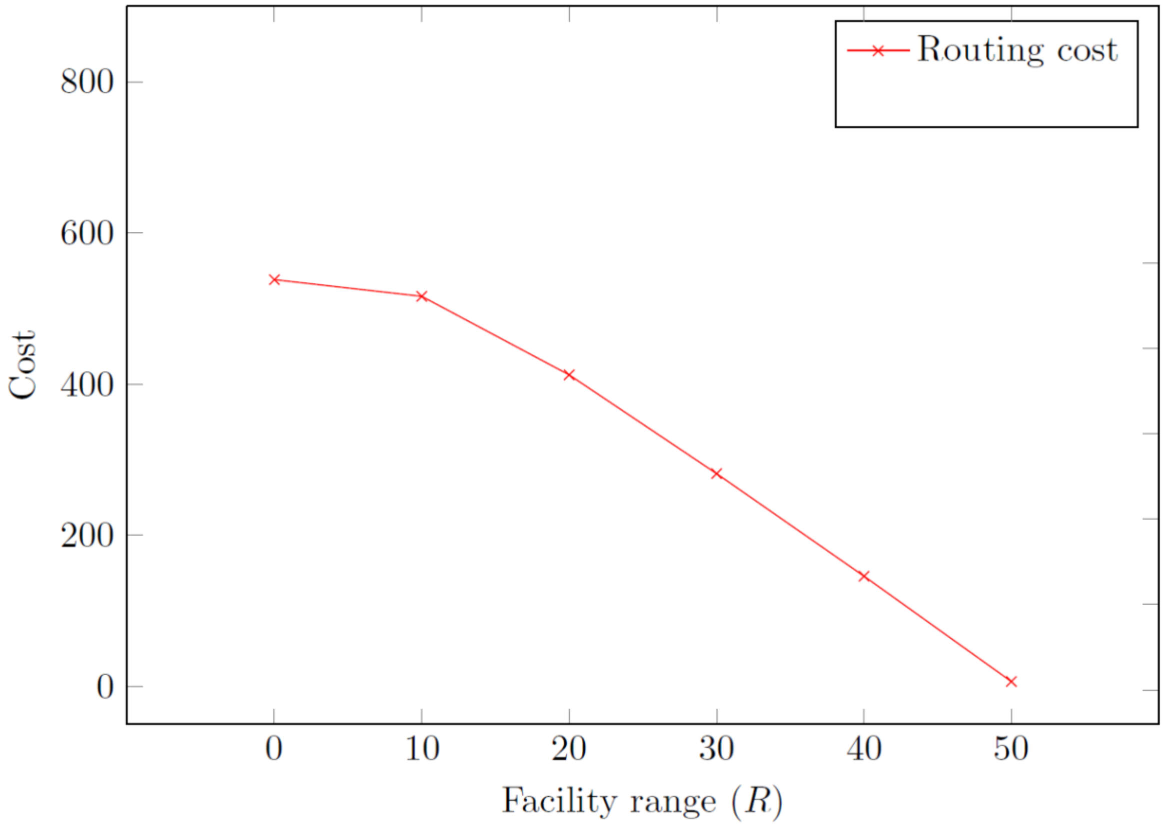
Solution times (s) for different T and R

Maximum route length (T)	Facility range (R)					
	0	10	20	30	40	50
100	4984.9	4976.5	3565.8	1237.1	219.0	19.6
125	5896.1	5855.2	4765.8	2153.0	983.9	400.8
150	6857.5	6688.1	5638.6	2903.1	1743.5	1047.5
Average	5912.8	5839.9	4656.7	2097.7	982.1	489.3

The location-or-routing problem

Computational results

- Impacts of facility coverage (R) on the cost.



The location-or-routing problem

Computational results

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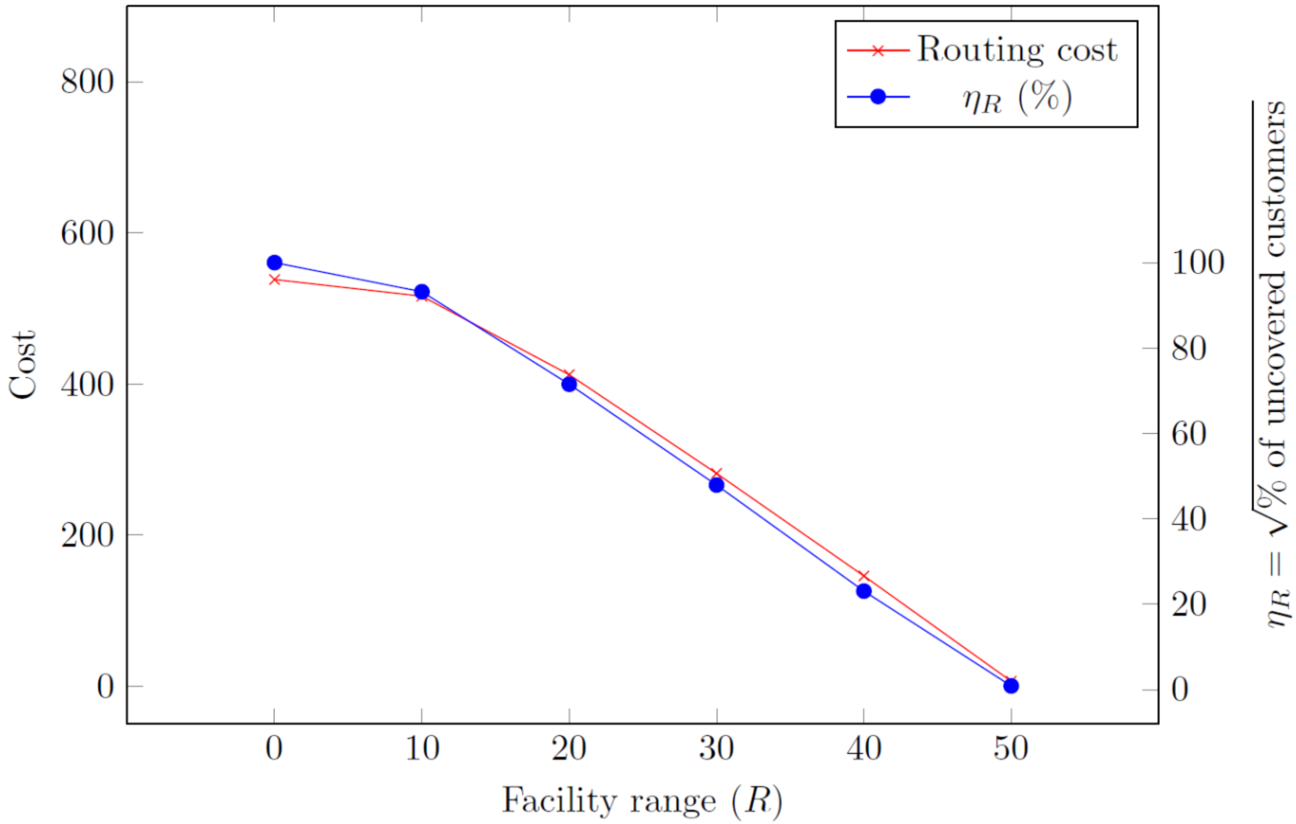


The location-or-routing problem

Asymptotic analysis

- Continuum approximation models (Beardwood et al., 1959; Daganzo, 1984a; 1984b).
- Given n points that are uniformly and independently distributed in an (fairly compact) area of size A, we have

$$\frac{L^*}{\sqrt{n}} \rightarrow \beta\sqrt{A} \;\text{ as }\; n \rightarrow \infty$$

where L^* is the optimal length in TSP and β is a constant term.

- An approximation for L^* is $\beta\sqrt{nA}$.
- The expected number of customers within a radius of R is

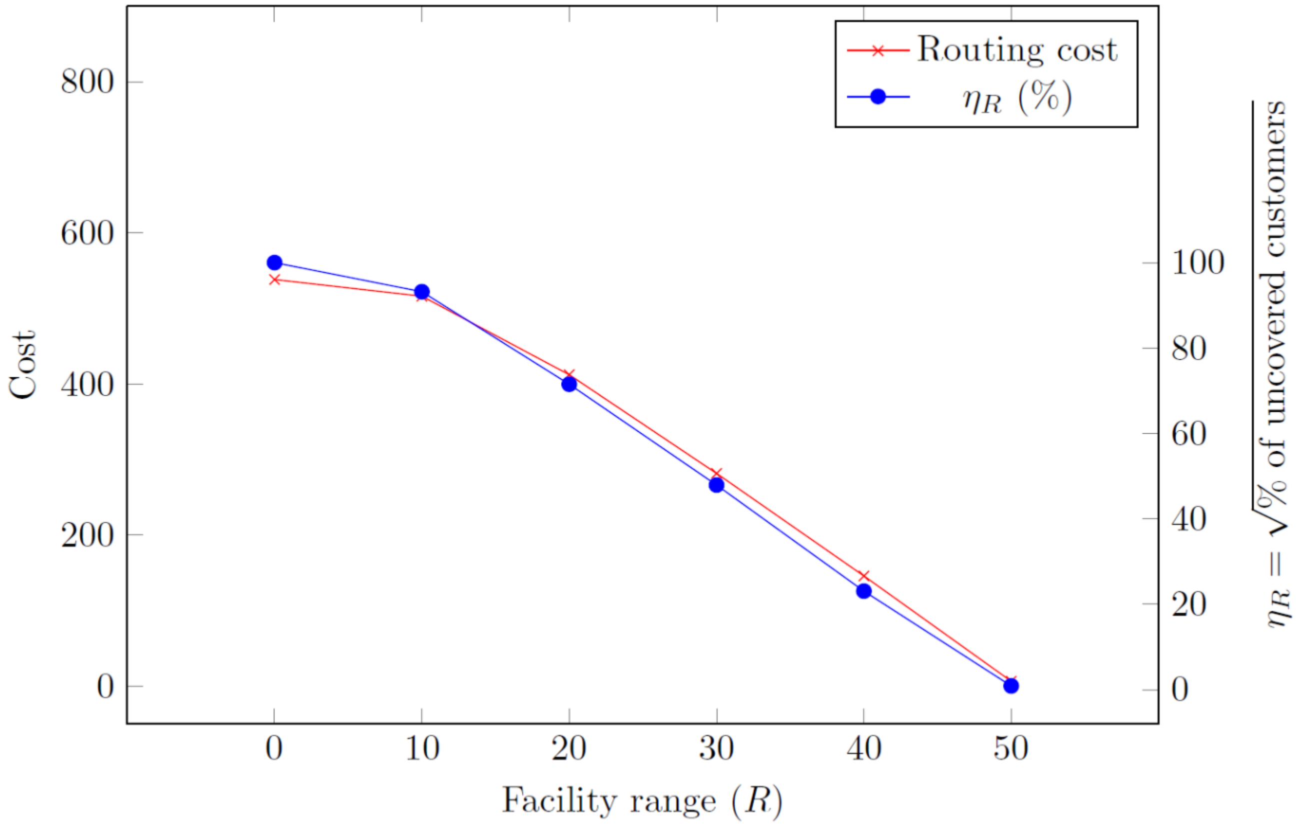
$$\left(\frac{\pi R^2}{A}\right)n.$$

- The cost changes linearly in $\sqrt{n}$, which changes linearly in R.

The location-or-routing problem

Computational results

- Impacts of facility coverage (R) on the cost.



Outline

- Service can be provided
  - at a facility or
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- Location-or-routing problem
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  - Branch and price algorithm.

Questions?

Maximum availability service facility location problem

- Testing and vaccination center
  - Walk-in
    - Median location problem
    - Covering location problem
  - Drive-thru
    - Flow capturing location problem
      - Structurally, it is the same as covering location problem

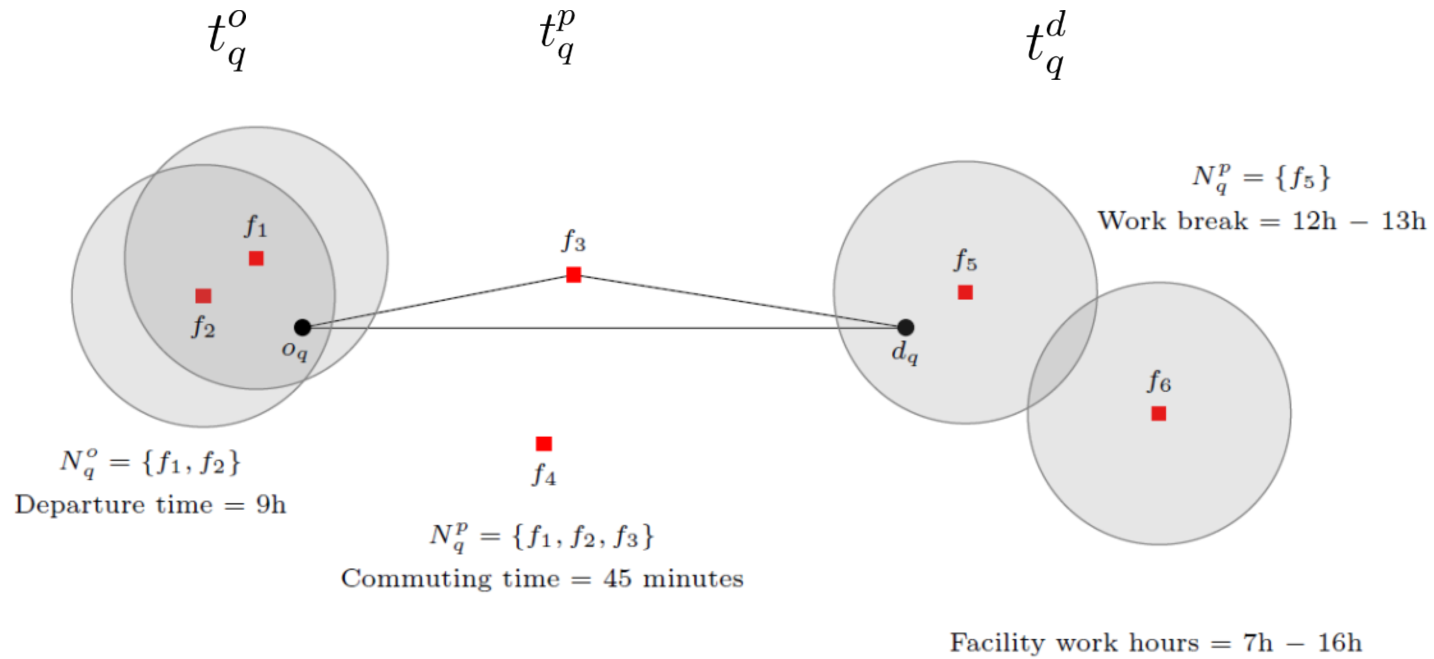
Source: Eduardo Contreras/Eduardo Contreras/The San Diego Union-Tribune / https://www.sandiegouniontribune.com/news/health/story/2020-04-15/county-shuts-down-pop-up-testing-center
https://bc.skipthewaitingroom.com/Content/Pictures/ClinicPictures/promoted-clinic-image.jpg

Maximum availability service facility location problem

- The keyword is the "availability"

- Capturing the demand for multiple times?
- Time dimension?
- Applications in
  - Testing and vaccination medical centers,
  - Government offices,
  - Polling/voting stations.

Maximum availability service facility location problem



- **Definition.** The problem is defined as selecting a subset of the facilities to open such that the total availability provided to the demand is maximized and the maximum contribution of each demand is at most t_{max}.

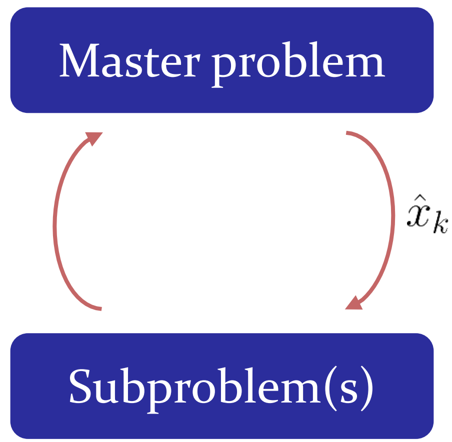
Maximum availability service facility location problem

Benders decomposition

$$\begin{aligned} \text{minimize} \quad & \eta \\ \text{subject to} \quad & \eta \le f(x) \\ & \sum_{k \in K} x_k = m \\ & x_k \in \{0, 1\} \end{aligned}$$

Analytical subproblem solution

Pareto	$\eta_q \le f_q^p(x)$	$\eta \le f^p(x)$
Non-Pareto	$\eta_q \le f_q(x)$	$\eta \le f(x)$
	Multicut	Singlecut



- The subproblem LP can be decomposed based on *q*.
- For each subproblem, cuts can be added separately.

} Multicut

- The subproblem LP is almost always degenerate.
- We can "select" a cut among the alternative opt.sol.s.

} Pareto-optimal cut

Maximum availability service facility location problem

Benders decomposition – Computational results

n	m	CPLEX	BD-Single (Subp. solved using LP)	BD-Single (Subp. solved analytically)
	5	4.4	1.0	0.2
100	10	7.7	1.0	0.2
	15	10.6	1.3	0.3
	5	65.5	13.8	1.7
200	10	95.9	12.3	1.6
	15	116.7	14.0	1.8
	5	337.0	55.3	5.2
300	10	477.3	66.0	5.9
	15	621.7	66.2	5.2
	5	949.7	170.0	12.5
400	10	1138.9	190.0	13.4
	15	1649.7	202.1	12.4

- The analytical solution is dominating.

Maximum availability service facility location problem

Benders decomposition – Computational results

Reduction in # cuts

%98 %47

n	m	CPLEX	BD-Single	BD-Multi	BD-single-pareto	BD-multi-pareto
	5	337.0	15.8	514.1	43.0	265.8
	10	477.3	29.3	568.7	77.9	310.9
	15	621.7	31.4	568.9	66.8	358.8
	20	739.2	162.9	675.6	108.7	546.9
300	30	1043.7	800.9	653.0	139.8	596.9
	40	1171.6	TL	754.6	286.2	650.6
	50	987.6	TL	709.1	240.8	602.4
	75	1003.5	TL	882.5	468.5	760.0
	100	840.9	TL	806.2	445.4	663.9

- The Pareto solution is dominating.
- TL: time limit (1h).

Maximum availability service facility location problem

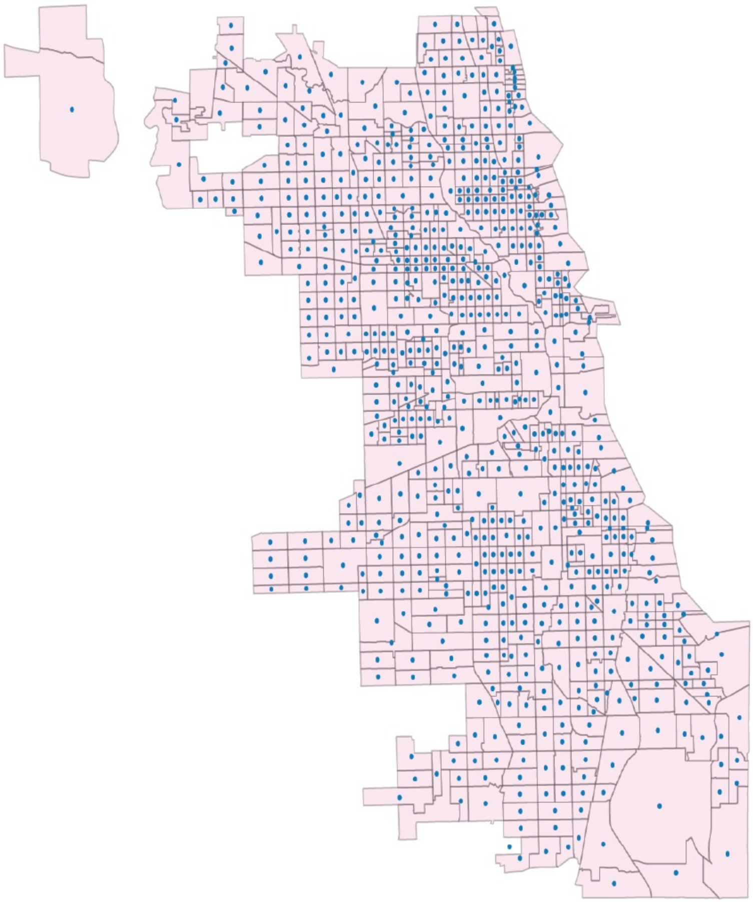
Benders decomposition – Computational results

m	n	Cplex	BD-multi-pareto	BD-single-pareto
	5	**TL**	**M**	1387.7
	10	**TL**	**M**	1388.2
	15	**TL**	**M**	1915.0
	20	**TL**	**M**	2463.6
1000	30	**TL**	**M**	2460.0
	40	**TL**	**M**	2420.0
	50	**TL**	**M**	3451.6
	75	**TL**	**M**	**TL**
	100	**TL**	**M**	**TL**

- TL: time limit (1h), M: memory

Maximum availability service facility location problem

Chicago case study

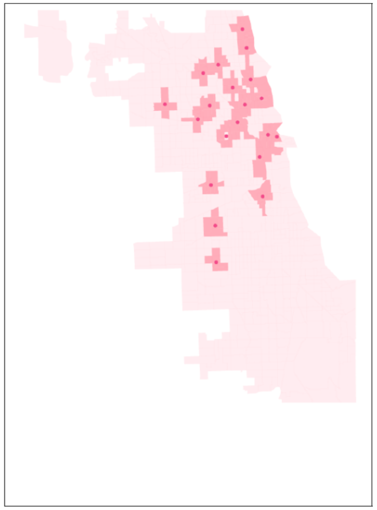


Population (million)	# Nodes	# OD pairs
2.701	797	35501

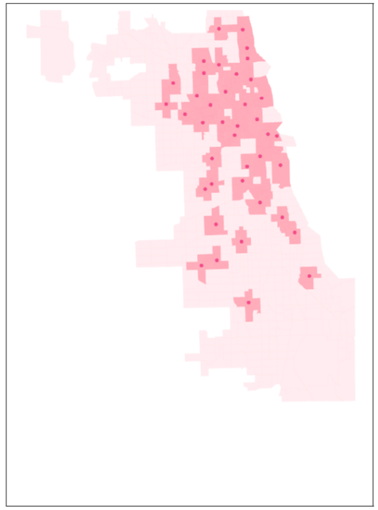
	Distance (km)	
Node Density (#/km^2)	minimum	maximum
1.353	0.19	62.1

Maximum availability service facility location problem

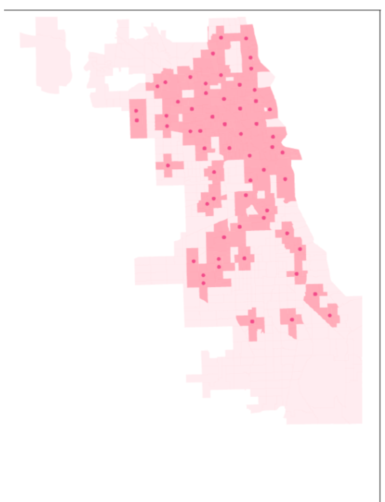
Chicago case study



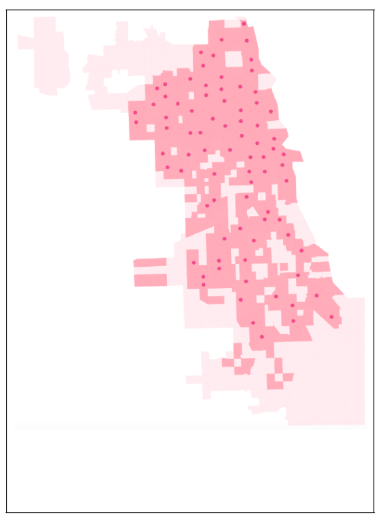
(a) Facilities = 20, Coverage = 32.08%



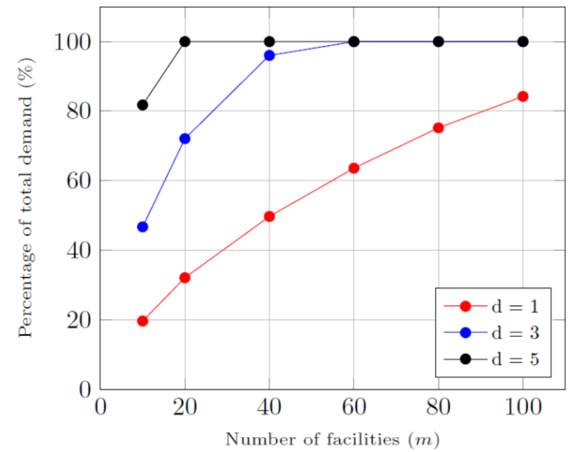
(b) Facilities = 40, Coverage = 49.70%



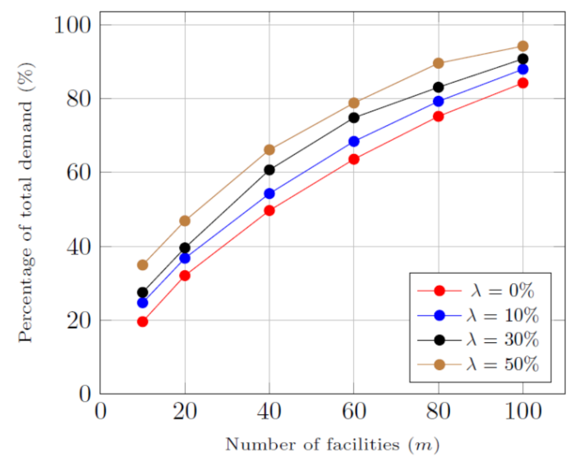
(c) Facilities = 60, Coverage = 63.59%



(d) Facilities = 80, Coverage = 75.19%



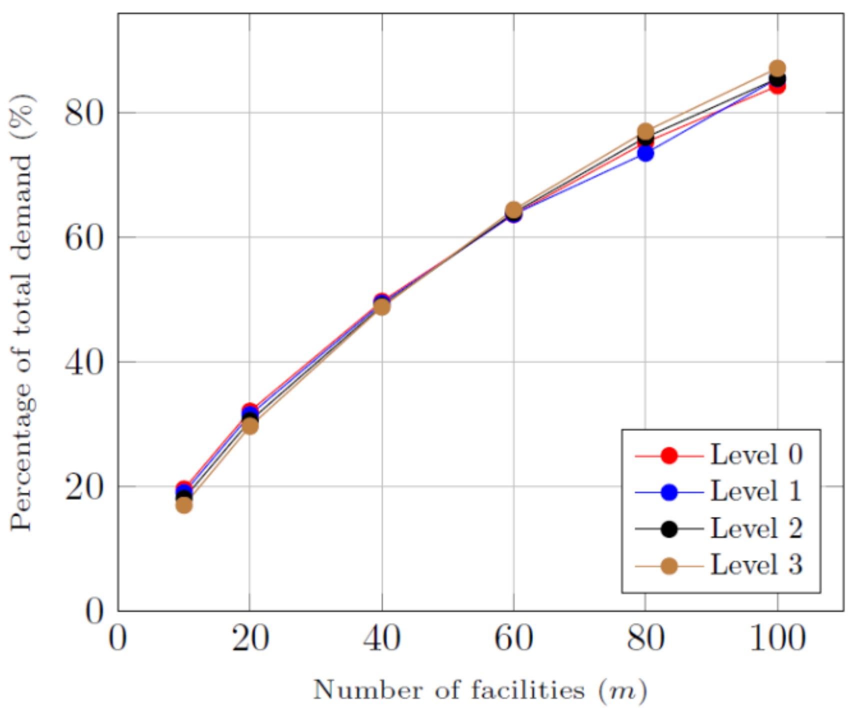
Percentage of total demand covered for Level 0 and $\lambda = 0$



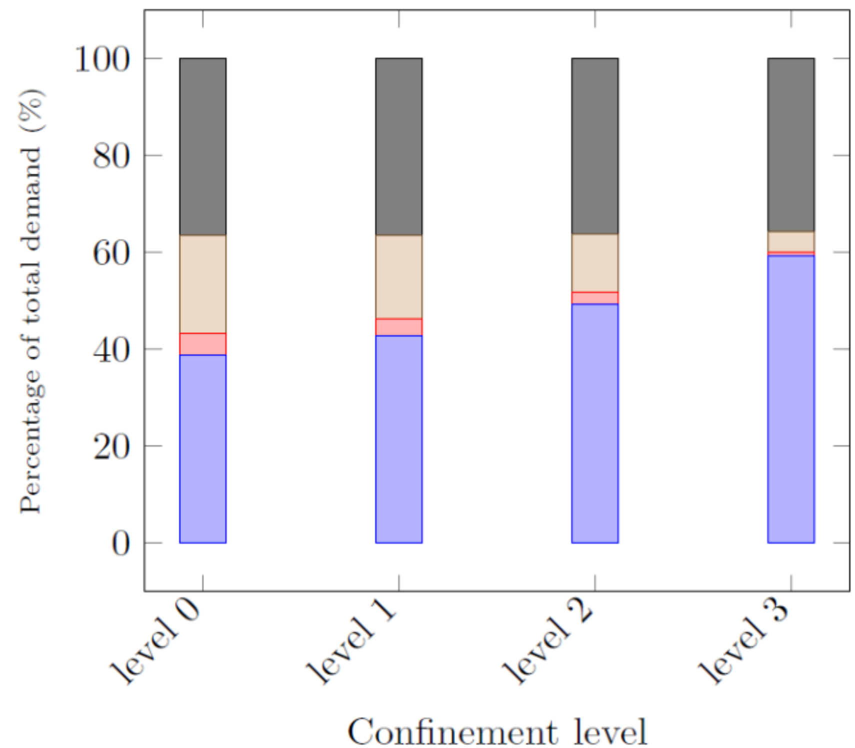
Percentage of total demand covered for Level 0 and $d = 1$

Maximum availability service facility location problem

Chicago case study



Percentage of total demand covered for $\lambda = 0\%$ and $d = 1$



Type of coverage for $m = 60$, $\lambda = 0\%$, and $d = 1$

Coverage at Origin | Coverage at destinantion | Coverage on path | Not covered

Maximum availability service facility location problem

Chicago case study

Optimization level	Actual confinement level: 0	1	2	3
0	0.0	3.1	5.7	8.1
1	1.7	0.0	2.9	4.8
2	2.8	2.1	0.0	2.7
3	4.2	2.9	1.1	0.0

Service facility location optimization during a pandemic

Future research topics

- *Vaccine logistics,*
- *The uncertainty,*
- *Social inequity:*
  - Pandemics does not impact people uniformly. The social inequalities in the society create hotspots for the virus to thrive.
- *Symptomatic testing:*
  - From the onset of the COVID-19 pandemic until June 2020, mainly the symptomatic patients were tested. However asymptomatic patients also exist, and their prevalence can be as high as 88% in certain groups.
- *Reactive decision-making.*

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