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Fifty years of operational research: 1972–2022

G. Laporte

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GERAD HEC Montréal
3000, chemin de la Côte-Sainte-Catherine
Montréal (Québec) Canada H3T 2A7

Tél. : 514 340-6053
Téléc. : 514 340-5665
info@gerad.ca
www.gerad.ca

Fifty years of operational research: 1972–2022

Gilbert Laporte [a]

[a] *GERAD & Department of Decision Sciences,*
HEC Montréal, Montréal (Qc), Canada, H3T 2A7

`gilbert.laporte@hec.ca`

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Abstract : In July 2022, I received the EURO Gold medal at the 32$^{\text{nd}}$ EURO Conference held in Espoo, Finland. This paper is based on the presentation I made at the medal award ceremony. It covers 10 topics on which I have worked throughout my career. These are the seriation problem, rotating work schedules, deterministic vehicle routing, stochastic vehicle routing, examination timetabling, districting, waste management, metro network design, green transportation, and humanitarian logistics.

Keywords : Combinatorial optimization, operational research, EURO Gold medal, applications

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1 Introduction

The EURO Gold medal is awarded when a EURO Conference is held to an individual or a group for an outstanding contribution of operational research. It is intended to reflect contributions that have withstood the test of time, and hence is awarded for a body of work rather than a single piece.[1] In July 2022, I received the EURO Gold medal at the 32nd EURO Conference held in Espoo, Finland. On this occasion I was asked to deliver a 30-minute presentation on my major and recent contributions to the body of knowledge in operational research. To this end, I have selected 10 topics that illustrate the breadth of my research interests and achievements spanning 50 years from the beginning of my Ph.D. studies in 1972 to 2022. These are the seriation problem, rotating work schedules, deterministic vehicle routing, stochastic vehicle routing, examination timetabling, districting, waste management, metro network design, green transportation, and humanitarian logistics. The present paper expands on my award ceremony presentation.

2 The seriation problem

From 1972 to 1975, I was a Ph.D. student at the London School of Economics (LSE) under the supervision of Ailsa H. Land who received the 2021 EURO Gold Medal. In a recent paper (Laporte, 2021) I provided a brief account of this period. My thesis (Laporte, 1975) focused on the study of the seriation problem. Liiv (2010) describes seriation as "an explanatory data analysis technique to reorder objects into a sequence along a one-dimensional continuum so that it best reveals regularity and patterning among the whole series". Seriation algorithms are rooted in combinatorial optimization (Seminaroti, 2016) and in statistical techniques such as multidimensional scaling (Kendall, 1969b; Rodgers and Thompson, 1992), and more recent developments based on data mining, information visualisation and network analysis (Liiv, 2010).

My thesis focused on an application of the seriation problem arising in archaeology, namely that of inducing a likely chronological ordering of graves based on artifacts they contain, such as pottery, religious symbols, jewels, etc. Robinson (1951) hypothesized that the importance of an artifact in a grave increases over time and attains a peak before declining. The relevant information can be summarized in the form of an $n \times m$ "abundance matrix" $A = (a_{ij})$, where a_{ij} is the absolute frequency of artifact j in grave i. Figure 1 depicts two abundance matrices, the first with an arbitrary grave ordering, and the second exhibiting a perfect ascending and descending pattern. Of course, such regular patterns are not always achievable. Doran and Powell (1972) worked with a simplified binary "incidence matrix" $E = (e_{ij})$, where $e_{ij} = 1$ if and only if $a_{ij} > 0$, i.e., artifact j is present in grave i. Then, in a perfect ordering of the rows of E, the 1s should be consecutive in each column (Figure 2). Again, a perfect ordering does not always exist, and then a natural problem is to determine a permutation of the rows of E that will minimize a "score" equal to the sum of the spreads of 1s over all columns, such as in the example of Figure 3. This is the version of the seriation problem I considered in my thesis and that I refer to as "the seriation problem" in this paper. This problem is not restricted to archaeological seriation, but also arises in diverse fields such as epigraphy (Ştefan, 1971), genetics (Fulkerson and Gross, 1965; Atkins et al., 1998), scheduling (Adelson et al., 1976), and VLSI design (Möhring, 1990).

An important property of a binary matrix E is whether or not it possesses the consecutive 1s property, i.e., whether there exists a permutation of the rows of E in which the 1s are consecutive in each column. Fulkerson and Gross (1965) have designed a polynomial algorithm based on interval graphs that determines whether a matrix satisfies this property and, if this is the case, identifies a permutation of the rows of E in which the 1s are consecutive in each column, i.e., the score is minimized. The seriation problem is NP-hard since it is equivalent to the NP-complete "consecutive 1s matrix augmentation problem": given a binary matrix E, determine whether there exists a matrix

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[1] https://en.wikipedia.org/wiki/EURO_Gold_Medal

$\tilde{E}$ derived from E by changing at most K entries equal to 0 into a 1 so that $\tilde{E}$ has the consecutive 1s property (Booth, 1975; Papadimitriou, 1976; Garey and Johnson, 1979).

Graves	Artifacts					
	1	2	3	4	5	6
1	1	3	5	4	4	0
2	0	7	0	0	1	5
3	1	0	0	3	3	0
4	0	3	4	1	5	0
5	3	3	7	6	9	0

(a) Unordered grave sequence

Graves	Artifacts					
	1	2	3	4	5	6
2	0	7	0	0	1	5
4	0	3	4	1	5	0
5	3	3	7	6	9	0
1	1	3	5	4	4	0
3	1	0	0	3	3	0

(b) Ordered grave sequence

Figure 1: Unordered and ordered abundance matrices.

Graves	Artifacts					
	1	2	3	4	5	6
1	1	1	1	1	1	0
2	0	1	0	0	1	1
3	1	0	0	1	1	0
4	0	1	1	1	1	0
5	1	1	1	1	1	0

(a) Unordered grave sequence

Graves	Artifacts					
	1	2	3	4	5	6
2	0	1	0	0	1	1
4	0	1	1	1	1	0
5	1	1	1	1	1	0
1	1	1	1	1	1	0
3	1	0	0	1	1	0

(b) Ordered grave sequence

Figure 2: Unordered and ordered incidence matrices corresponding to the abundance matrices of Figure 1.

Graves	Artifacts					
	1	2	3	4	5	6
1	1	1	1	1	1	0
2	0	1	1	0	0	0
3	0	0	1	0	1	0
4	1	0	0	1	0	1
5	1	1	0	1	1	1

(a) Unordered incidence matrix

Graves	Artifacts					
	1	2	3	4	5	6
2	0	1	1	0	0	0
4	1	0	0	1	0	1
5	1	1	0	1	1	1
1	1	1	1	1	1	0
3	0	0	1	0	1	0

(b) Ordered incidence matrix of least score

Figure 3: Unordered and ordered incidence matrices. In Figure 3a the score is equal to $19(= 4 + 4 + 2 + 4 + 4 + 1)$. In Figure 3b the score is minimized and is equal to $14(= 2 + 3 + 4 + 2 + 2 + 1)$.

The seriation problem is notoriously difficult to solve exactly. Two natural algorithms are dynamic programming and mixed integer linear programming (MILP). As noted in Adelson et al. (1976) and Laporte (1987), the complexity of the dynamic programming algorithm is $O(mn2^n)$, and therefore only relatively small instances can be solved using this technique. The first MILP formulation for the seriation problem is due to Doran and Powell (1972). Let x_i be the position in the permuted matrix E' of row i of E, and let u_j and v_j be the row numbers of the first and last non-zero entries in column j of E'. The problem can then be formulated as follows:

$$\text{minimize} \sum_{j=1}^{m} (v_j - u_j) \tag{1}$$

subject to

$$\sum_{i=1}^{n} x_i = n(n+1)/2 \tag{2}$$

$$|x_k - x_l| \geq 1 \qquad (k < l; k, l = 1, \ldots, n) \tag{3}$$

$$v_j \geq x_i \qquad (e_{ij} = 1) \tag{4}$$

$$u_j \leq x_i \qquad (e_{ij} = 1) \tag{5}$$

$$u_j, v_j \geq 1 \qquad (j = 1, \ldots, m) \qquad (6)$$

$$1 \leq x_i \leq n \qquad (i = 1, \ldots, n). \qquad (7)$$

In this formulation, the objective minimizes the score. Constraints (2) and (3) ensure that the x_i variables describe a permutation of $(1, \ldots, n)$ and therefore there is no need to declare these variables as integers. Constraints (4) and (5) link the x_i, u_j and v_j variables. Constraints (6) and (7) define the domains of the variables. Constraints (3) can be linearized as

$$\begin{aligned} x_k - x_l + n\delta_{kl} &\geq 1 \\ x_k - x_l + n\delta_{kl} &\leq n - 1 \\ \delta_{kl} &\in \{0, 1\} \qquad (k < l; k, l = 1, \ldots, n), \end{aligned}$$

or branching can be performed dynamically on the alternative $x_k - x_l \geq 1$ or $x_k - x_l \leq -1$. In either case, the lower bound obtained by initially relaxing constraints (3) will be weak. As a result, it will be difficult to solve even medium-size instances exactly using this model.

It is possible to derive a lower bound on the optimal score by solving an auxiliary traveling salesman problem (TSP), the advantage being that in practice the TSP is much easier to solve than the seriation problem. To this end, first observe that one should favour solutions in which any two consecutive rows of E' both have a 1 in as many columns as possible. Hence define a dissimilarity coefficient $c_{kl} = m - \sum_{j=1}^{m} e_{kj} e_{lj}$ between rows k and l of E. A traveling salesman instance with costs c_{kl} is obtained by introducing in E a dummy row of 0s in position $n + 1$. The TSP solution will be such that the total number N_j of sequences of 0s between two 1s over all columns j will be minimized, but not the sum of their lengths L_j. For example, the matrix of Figure 3b contains two such sequences of 0s, but their total length is three. Hence, the TSP will minimize $\sum_{j=1}^{m} \left(N_j - 1 + \sum_{i=1}^{n} e_{ij}\right)$, which is a lower bound on the score $\sum_{j=1}^{m} \left(L_j - 1 + \sum_{i=1}^{n} e_{ij}\right)$. The quality of the TSP lower bound is directly measured by $\sum_{j=1}^{m} \left(L_j - N_j\right)$.

Note that the minimization of a dissimilarity function has long been advocated by archaeologists (see, e.g., Brainerd (1951); Robinson (1951); Kendall (1969a,b); Wilkinson (1971)). However, Laporte (1976) observed that minimizing the score rather than the dissimilarity function just defined seems to yield orderings that are closer to those proposed by archaeologists on real data such as those of the 63-row E matrix of the La Tène Cemetery (Hodson, 1968).

Given the fact that the seriation problem is very difficult to solve exactly and the data are often imperfect, a practical solution methodology is to apply a row interchange local search heuristic similar to those commonly applied to the TSP. Successful implementations of such heuristics are reported in Laporte and Taillefer (1987) and in Gendreau et al. (1994).

3 Rotating work schedules

My interest in rotating work schedules originates from a consultancy project undertaken for the Montreal police union together with my colleague Jean-Marc Rousseau, soon after my Ph.D. studies. A rotating work schedule consists of a cyclic arrangement of work shifts, typically day (D), evening (E) and night (N), and off-duty periods (X). Figure 4 depicts a five-week cycle (Montreal police union) and a 12-week cycle (some employees of the Quebec Department of Transportation monitoring traffic on closed-circuit TV). The number of shifts of each type on any day of the week reflects the distribution of workers among the shifts needed to meet the workload requirements. Thus, in Figure 4a, which is the schedule that was proposed for the Montreal police union, the distribution of workers among the shifts is uniform throughout the week, whereas in Figure 4b it is irregular, with a higher workload from Monday to Friday than during the weekend. One quarter of the workforce works on the night shift from Monday to Wednesday, whereas this proportion is only 1/12 from Thursday to Sunday. Rotating

Week	Mo	Tu	We	Th	Fr	Sa	Su
1	X	X	X	D	D	D	D
2	X	X	E	E	E	X	X
3	D	D	D	X	X	E	E
4	E	E	X	X	N	N	N
5	N	N	N	N	X	X	X

(a) A five-week cycle. Montreal police.

Week	Mo	Tu	We	Th	Fr	Sa	Su
1	D	D	X	D	D	X	X
2	D	D	D	D	D	X	X
3	X	X	X	E	E	E	E
4	E	E	X	X	E	E	E
5	E	E	E	X	X	X	X
6	D	D	D	X	X	D	D
7	D	D	D	D	D	X	X
8	E	E	E	E	X	E	E
9	E	E	E	E	E	X	X
10	N	N	N	N	N	X	X
11	N	N	N	X	X	N	N
12	N	N	N	X	X	D	D

(b) A 12-week cycle. Quebec Department of Transportation

Figure 4: Two rotating work schedules. D=day, E=evening, N=night, X=off.

work schedules are common in police and fire departments, and in factories that operate around the clock. All employees rotate through the same schedule in a cyclic fashion.

Prior to constructing a rotating work schedule, it is necessary to define a workload matrix that will specify the distribution of the workforce over the week. Figure 5 provides the workload matrices corresponding to the two schedules of Figure 4. Denote by d_i, e_i and n_i the number of weeks where work must be carried out during the day, during the evening and during the night, respectively on day i, and by x_i the number of weeks in which the employees are off work on day i. If c is the cycle length, then these values must satisfy the equality $d_i + e_i + n_i + x_i = c$ for all i. Moreover, if h is the duration of a work shift, then the average number w of working hours per week is $w = h\sum_{i=1}^{7}(d_i + e_i + n_i)/c$. Thus, if $h = 8$, then $w = 33.6$ for the schedule of Figure 4a and $w = 38$ for that of Figure 4b. In practice, determining an acceptable workload matrix, together with suitable values for c and h may require some negotiations between employers and employees. Short cycles are usually preferable to long ones since they are easier to administer. In addition, even if there are three work shifts, as is usually the case, the value of h may exceed eight since in some circumstances overlaps between shifts may be deemed desirable, which offers extra flexibility. For example, in our project with the Montreal police union, we worked with $h = 8.5$, which led to an average of $w = 35.7$ hours of work per week.

Week	Mo	Tu	We	Th	Fr	Sa	Su
d_i	1	1	1	1	1	1	1
e_i	1	1	1	1	1	1	1
n_i	1	1	1	1	1	1	1
x_i	2	2	2	2	2	2	2

(a) A five-week cycle. Montreal police.

Week	Mo	Tu	We	Th	Fr	Sa	Su
d_i	4	4	3	3	3	2	2
e_i	4	4	3	3	3	3	3
n_i	3	3	3	1	1	1	1
x_i	1	1	3	5	5	6	6

(b) A 12-week cycle. Quebec Department of Transportation

Figure 5: Two workload matrices

The next step is to construct the work schedule which must obey certain basic principles. Preferably, a shift change may only occur after a day off, and long work stretches should be followed by long off stretches. Upper and lower bounds are imposed on the number of consecutive work or rest periods, usually between two and seven. The number of achievable weekends off should ideally be reached, and these should be regularly spaced, as much as possible. As a rule, forward rotations (day, evening, night) are more desirable than backward rotations (day, night, evening) (Rosa and Colligan, 1997) in terms of ergonomy. It is also often suggested not to have too many night periods in a row (Schwarzenau et al., 1986; Knauth, 1996; Rosa and Colligan, 1997), but in all organizations I have worked with, the employees preferred having long uninterrupted night sequences, up to seven nights in a row, like in the example of Figure 4a. In practice, some of the above rules can be bent in order to achieve good solutions (Laporte, 1999).

What started as consultancy work soon evolved into a scientific project. While small schedules can easily be constructed by hand, it is often preferable to resort to a systematic algorithm for their construction, especially for tightly constrained instances or when it is desirable to quickly generate several candidate solutions. Integer linear programming models in which the variables correspond to all acceptable stretches of on-days and off-days have been proposed (Laporte et al., 1980), but with a limited success. Similarly, network flow models have been used (Balakrishnan and Wong, 1990), but these are typically unable to accommodate most side constraints encountered in practice. Constraint programming appears to be the most suitable method for the construction of rotating work schedules, especially in view of the fact that this is essentially a feasibility problem with no well-defined objective. Two such implementations are those of Musliu et al. (2002) and of Laporte and Pesant (2004). The latter paper describes several customized operators and reports very fast solution times on nine real-world instances that are very diverse in terms of requirements.

4 Deterministic vehicle routing

During my Ph.D. years at LSE, I became interested in the TSP partly because I used it in my thesis, but mostly because it was the main Ph.D. topic of my colleague Panagiotis Miliotis and a topic of interest of my thesis director who both made several key contributions to this problem (Laporte, 2021). After my studies I became interested in extending some of my TSP knowledge to the vehicle routing problem (VRP).

The VRP is one of the most studied problems in combinatorial optimization. It was introduced under the name “The Truck Dispatching Problem” by Dantzig and Ramser (1959) and renamed “The Vehicle Routing Problem” by Christofides (1976). In the undirected capacity constrained VRP, which is the version considered in this paper, there are n customers and a depot. A fleet of m identical vehicles of capacity Q is based at the depot, and each customer has a non-negative delivery demand $q_i \leq Q$. The value of m may be an input parameter of the problem or a decision variable, in which case a cost f is associated with it. The travel cost between i and j is equal to c_{ij}. It is usually equal to either the distance or the travel time between i and j. The VRP consists of designing m vehicle routes of least total cost, such that each route starts and ends at the depot, each customer is visited once by exactly one vehicle, and the vehicle capacity on any route is not exceeded. Two other common classes of constraints are distance constraints stating that the length of any route should not exceed a limit L, and a time window $[a_i, b_i]$ imposed on the start of service at each customer i.

Multiple other constraints exist, reflecting the numerous practical applications of the problem. For example, the vehicle fleet may be heterogeneous, which is more often than not the case in practice; vehicles may have different compartments for different kinds of products; the vehicle may perform pickups and deliveries, simultaneously or not; there may be several depots at which cross-docking operations sometimes take place; customers may have multiple or soft time windows; some customers may be optional, in which case visiting them may generate a profit; some parameters of the problems may be stochastic; the problem may have to be solved in a dynamic fashion, etc. The edited book of Toth and Vigo (2014) presents an extensive treatment of these and other extensions.

The classical formulation for the symmetric TSP introduced by Dantzig et al. (1954) is defined on an undirected graph $G = (V, E)$, where $V = \{1, \ldots, n\}$ is the vertex set and $E = \{(i, j) : i, j \in V, i < j\}$ is the edge set. A cost c_{ij} is associated with each edge (i, j), and the binary variable x_{ij} is equal to 1 if and only if edge (i, j) is used in the solution. The TSP is formulated as follows:

$$\text{minimize} \sum_{(i,j)\in E} c_{ij} x_{ij} \tag{8}$$

subject to

$$\sum_{i<k} x_{ik} + \sum_{j>k} x_{kj} = 2 \qquad (k \in V) \tag{9}$$

$$\sum_{i,j \in S} x_{ij} \leq |S| - 1 \qquad (S \subset V, |S| \geq 3) \tag{10}$$

$$0 \leq x_{ij} \leq 1 \qquad (i, j \in V, i < j) \tag{11}$$

$$x_{ij} \text{ integer} \qquad (i, j \in V, i < j). \tag{12}$$

The objective function (8) minimizes the cost of the traveling salesman tour, constraints (9) are degree constraints, constraints (10) are subtour elimination constraints, and constraints (11) and (12) define the domains of the variables. If branch-and-cut is applied to the solution of the problem, constraints (10) and (12) are initially relaxed and dynamically imposed as they are found to be violated. In his seminal algorithms, Miliotis (1978) makes use of Gomory cuts to reach integrality.

My first contribution to the VRP was to extend the work of Miliotis (1976) to the VRP. In the VRP formulation, V is redefined as $V = \{0, \ldots, n\}$, where vertex 0 is the depot, $V \setminus \{0\}$ is the set of customers, and x_{ij} indicates the number of times edge (i, j) is used in the solution. The formulation is then:

$$\text{minimize} \sum_{(i,j) \in E} c_{ij} x_{ij} + fm \tag{13}$$

subject to

$$\sum_{j \in V \setminus \{0\}} x_{0j} = 2m \tag{14}$$

$$\sum_{i<k} x_{ik} + \sum_{j>k} x_{kj} = 2 \qquad (k \in V \setminus \{0\}) \tag{15}$$

$$\sum_{i,j \in S} x_{ij} \leq |S| - \lceil \sum_{i \in S} q_i / Q \rceil \qquad (S \subset V \setminus \{0\}, |S| \geq 3) \tag{16}$$

$$x_{0j} \in \{0, 1, 2\} \qquad (j \in V \setminus \{0\}) \tag{17}$$

$$x_{ij} \in \{0, 1\} \qquad (i, j \in V \setminus \{0\}, i < j). \tag{18}$$

The constrainsts have the same interpretation as those of the TSP. In the generalized subtour elimination constraints (16), the second term of the right-hand side is a lower bound on the number of vehicles necessary to serve all customers of S.

Laporte and Nobert (1983) have developed a branch-and-cut algorithm based on this model for the VRP with capacity constraints, and Laporte et al. (1985) later extended this algorithm to the VRP with capacity and route length constraints. These algorithms were successful in solving loosely constrained instances of the problems. Laporte and Nobert (1984) were the first to generalize the Chvátal (1973) comb inequalities to the VRP. These early results were later extended by several other researchers, namely through the development of new families of comb inequalities and of other valid inequalities. For surveys on this topic, see Toth and Vigo (2002) and Naddef and Rinaldi (2002). By and large, the efficacy of branch-and-cut algorithms for the solution of the VRP is limited. Nowadays, the best algorithms are based on branch-and-cut-and-price which combines branch-and-cut and column generation. The best such implementations are currently those of Poggi and Uchoa (2014), Pecin et al. (2014, 2017), and Sadykov et al. (2020).

Because exact algorithms are not sufficiently powerful and robust to solve large VRP instances, especially in view of the fact that most real-life applications include numerous complicating features, heuristics are commonly used in practice. The best available heuristics for the VRP are mostly based on the hybridization of several metaheuristic principles, namely on the combination of mathematical programming with large neighbourhoods, and on decomposition or cooperation schemes. In addition, there is now a tendency to develop heuristics that can handle several VRP features using the same algorithmic structure and parameters (Vidal et al., 2014). For overviews and recent experiments, see Laporte et al. (2014), Accorsi and Vigo (2021), and Santini et al. (2023).

A recent trend in the development of VRP heuristics is the inclusion of emerging objectives, such as those described in Vidal et al. (2020). These include 1) profitability (performance ratios, profits, outsourcing); 2) service quality (cumulative objectives, inconvenience measures, service levels); 3) equity (workload balance, service equity, collaborative planning); 4) consistency (temporal, person-oriented, regional); 5) simplicity (compactness, separation, navigation complexity); 6) reliability (expected cost or loss, probability of failure); 7) externalities (emissions, safety risk).

5 Stochastic vehicle routing

Three major sources of uncertainty in the VRP are the demands, travel and service time durations, and the presence of customers (Gendreau et al., 1995b, 2016; Oyola et al., 2017, 2018). The VRP with stochastic demands is the most studied case and is also the problem that first drew my interest. The chance-constrained version of this problem is relatively simple. It suffices to replace the right-hand side of constraints (16) in the deterministic VRP model with $|S| - m_\alpha(S)$, where $m_\alpha(S)$ is equal to the minimum number of vehicles needed to serve all customers of S so that the total demand of S exceeds $m_\alpha(S)Q$ with a probability at most equal to α. In other words, if ξ_i is the demand of customer i, then $m_\alpha(S)$ is the smallest integer such that $P(\sum_{i \in S} \xi_i > m_\alpha(S)Q) \leq \alpha$. It is easy to compute the value of $m_\alpha(S)$ if the ξ_i random variables are independently distributed and follow an additive distribution such as normal or Poisson.

Most authors who have worked on the VRP with stochastic demands have developed recourse models rooted in the a priori optimization paradigm (Bertsimas et al., 1990): a first-stage solution is first computed, a random event occurs which may render this solution infeasible, and a recourse action which generates a penalty is then applied. The aim is to compute a first-stage solution such that its cost, plus the expected cost of recourse, is minimized. In the VRP with stochastic demand the random event is the realization of the customer demands. The most common recourse action is where each vehicle follows its route as planned until its capacity becomes exceeded, returns to the depot to replenish, and resumes its planned route at the point of failure. Other recourse actions are possible, such as reoptimizing the remaining portion of the route at each point of failure, or making preventive returns to the depot in anticipation of a higher recourse cost in the remaining part of the route (Dror et al., 1989). Dror et al. (1993) assume that a route failure can only occur at one of the last k customer locations of the planned route, which is realistic in many contexts for small values of k. The problem can then be solved as a sequence of k TSPs, and therefore reduces to a single TSP if $k = 1$. Lei et al. (2011) have designed an adaptive large neighbourhood heuristic for the VRP with stochastic demands and time windows. The recourse action is a back-and-forth trip to the depot, which increases the route duration and therefore triggers a chain reaction on the arrival times at the customers on the planned route. The problem is solved under the assumptions that at most one failure per route may occur and that the customers whose time window is missed will be served by a special delivery which generates an extra cost. Another interesting case of the VRP with stochastic demands is where the expected cost of recourse is bounded by a given percentage β of the first-stage solution cost. Laporte et al. (1989) show that this case can be modeled and solved as a deterministic VRP.

In the VRP with stochastic travel durations, the random variables are usually not independent, which severely complicates the computations. For example, one needs to resort to convolutions to compute the probability distribution of route durations. One realistic way of surmounting this difficulty is to work with probabilistic scenarios, for example low, medium or heavy traffic simultaneously affecting all travel durations in the same way (Laporte et al., 1992). Such problems are defined with a limit L on the duration of a vehicle route, and usually an overtime cost is incurred whenever a route duration exceeds L. This is also the case in problems with random service times (Lei et al., 2012), but there it makes sense to assume that the service times are independent random variables, which simplifies the computations. One interesting exception to the overtime penalty cost is found in the paper by Lambert et al. (1993) which deals with money collection at bank branches. Here, any money arriving after a given deadline loses one day's interest, and the cost of recourse is equal to the total lost interest.

The study of routing problems with stochastic customers is rooted in the work of Jaillet (1985) for the TSP, and of Bertsimas (1992) for the VRP. In these problems vehicle routes are first designed and the list of absent customers is revealed just before the routes are followed, but the routes cannot be changed, except for the fact that the absent customers are skipped. Contrary to what happens in the stochastic VRPs just described, skipping the absent customers generates a saving, in other words a negative recourse cost. Laporte et al. (1994) have published the first exact algorithm for the TSP with stochastic customers. Gendreau et al. (1995b, 1996) have designed a heuristic and an exact algorithm for the VRP with stochastic customers and demands. Note that in this case the recourse function includes a non-positive term due to absent customers, and a non-negative term due to stochastic demands.

The integer L-shaped method (Laporte and Louveaux, 1993) applies to stochastic integer programs with complete recourse. It was designed with stochastic VRPs in mind but is also of general applicability. This exact branch-and-cut algorithm is an extension of Benders decomposition (Benders, 1962) to stochastic programming. The method first computes a lower bound θ on the expected cost of recourse associated with the first-stage solution x of a subproblem, and it then computes the actual value of the recourse cost $Q(x)$ associated with x. If $Q(x) > \theta$, an optimality cut is added to force the generation of a new first-stage solution. Otherwise the optimum has been reached for the current subproblem. In its pure form this method can be quite weak, but it can be strengthened through the generation of valid inequalities at non-integer solutions, called lower bounding functionals (see, Laporte and Louveaux (1993); Laporte et al. (2002); Jabali et al. (2014)). The integer L-shaped method was applied to the solution of the VRP with stochastic demands (Laporte et al., 2002). In 2004, this work won the Best Paper Prize of the Transportation and Logistics Section of INFORMS. Generalizations (Carøe and Tind, 1998) and improvements (Angelo et al. (2016); Hoogendoorn and Spliet (2023)) of the integer L-shaped method have since been proposed.

6 Examination timetabling

Examination timetabling is a problem encountered in many colleges and universities and has attracted the attention of several researchers (see, e.g., Carter (1986), Carter and Laporte (1995), Qu et al. (2009), and McCollum et al. (2012)). In several teaching institutions, exams take place during the normal teaching periods and hence their timetabling does not pose any difficulty. Here we are interested in the case where exams are held in specific periods in mid-term or after the regular teaching activities. The exams are scheduled in a grid that specifies a set of periods during which the exams can take place. For example, if the exam period extends from Monday to Saturday and there are three allowable periods each day (morning, afternoon, evening), then the grid will consist of 18 periods. The basic problem is to assign exams to periods in such a way that there are no conflicts between exams, i.e., occurrences of students having to take two exams simultaneously. The number of periods required to schedule all exams is therefore at least as large as the size of the largest clique in the conflict graph. In practice, this number is much larger because of the presence of several side constraints. Also, as will be explained, examination scheduling is not only a feasibility problem.

The most common side constraints are as follows (Laporte and Desroches, 1984; Carter and Laporte, 1995; Carter et al., 1996): 1) Some exams may be preassigned to specific periods or groups of periods. For example, the exams of evening courses are often scheduled in the evening. 2) Similarly, some exams must be assigned to a specific room, such as a laboratory, a gymnasium or a concert hall. 3) There are constraints on room availability or on the total seating capacity. 4) Certain exams must be held in a specific sequence. For example, in some science courses, the theoretical exam is often scheduled before the laboratory exam. 5) In some cases where several exams correspond to courses given by the same instructor, it may be preferable to assign these exams to nearby rooms.

Two objectives are often considered in examination timetabling. 1) The most common objective is to space out the exams taken by the same student as much as possible. As in Carter et al. (1994) and Carter et al. (1996), this is expressed in the form "no x in y", to control how many times a student takes x exams in y consecutive periods. Therefore, one can simply impose hard constraints of the form "no x in y" for some x and y values (e.g., $x = y > 1$, $x = 2$ and $y = 3$), or soft constraints through penalties in the objective function. 2) In addition, there are sometimes preferred periods for some exams, which may be expressed as part of the objective.

While the examination timetabling problem can in principle be formulated as a mathematical program, this approach is impractical because of the large size of most realistic instances and because some "no x in y" constraints require large numbers of logical binary variables. For this reason, the problem is usually solved by means of local search heuristics in two steps: initial solution construction, and solution improvement by means of an exchange mechanism. In order to construct an initial solution, the exams are first ordered in a waiting list according to a given criterion, and then inserted one at a time in the grid while maintaining feasibility and selecting at each step the assignment that will least increase the objective function value. The waiting list may be reordered dynamically as the exams are scheduled. At some point it may prove impossible to feasibly schedule a new exam taken from the waiting list, in which case an already scheduled exam is bumped from the partially constructed schedule and reinserted in the waiting list of unscheduled exams so as to enable the scheduling of the new exam. Laporte and Desroches (1984) describe several ways of implementing the bumping mechanism in order to avoid cycling, such as imposing an upper limit on the number of times the same exam can be bumped from the partially constructed timetable. When this limit is reached, the instance is deemed infeasible. In numerous tests conducted on real and artificial instances, such situations have been extremely rare. A notorious example is that of Purdue University where, in 1993, 2,419 exams, 30,032 students, and 120,690 student-examinations had to be scheduled in 30 two-hour periods over five days. This proved impossible even if the density of the conflict matrix was only 0.03. Relaxing the no conflict constraints, but imposing a very high cost on them, yielded a timetable with some, but very few students in conflict (Carter et al., 1996). In practice, unplanned conflicts may arise for a variety of reasons, such as medical problems, in which case the universities implement ad hoc solutions to handle such situations.

Carter et al. (1996) have conducted an in-depth study of the best way to order the exams in the initial waiting list using five rules reflecting the difficulty of scheduling an exam: 1) largest number of conflicting examinations, 2) number of periods in conflict with the exam (as in Brélaz (1979) for the graph colouring problem), 3) number of students in conflict, 4) number of students enrolled for the exam, 5) random order. It was found that for loosely constrained instances, the first four rules yield similar results, but for tightly constrained instances, the third rule is preferable. The random order rule always fared the worst.

In order to space out exams evenly for most students, Laporte and Desroches (1984) applied decreasing weights w_s to the number of students enrolled in two exams separated by s periods, with $w_1 = 16$, $w_2 = 8$, $w_3 = 4$, $w_4 = 2$, and $w_5 = 1$, in a context where there are three periods per day, the evening of a given day and the morning of the following day being considered as consecutive. Carter and Laporte developed the EXAMINE examination scheduling software (described in (Carter et al., 1994)), which was tested or implemented in several universities worldwide. EXAMINE extends the work of Laporte and Desroches (1984). Its distinguishing feature lies in the implementation of a combination of "no x in y" conditions selected by the user, either as constraints or as objective components with appropriate weights. This enhancement has a major consequence in terms of computer implementation because it requires keeping information on every individual student, as opposed to simply recording the number of students common to two exams, as was the case in Laporte and Desroches (1984). For the development of EXAMINE and its implementation in several universities, Carter and Laporte ranked first in the Practice Prize competition of the Canadian Operational Research Society (CORS) in 1992.

Nowadays, many universities use sophisticated global data processing systems, such as PeopleSoft, to help manage their administrative activities. These systems often incorporate course scheduling and examination timetabling functionalities, in which case the need for dedicated examination timetabling software no longer exists.

7 Districting

The purpose of districting is to partition a territory into districts that satisfy some properties. Kalcsics and Ríos-Mercado (2019) identify four main application areas: political districting, sales territory design, service districting, and distribution districting. I have conducted research in each of these areas.

Political districting is probably the most common case. The aim is to construct political constituencies that satisfy some basic criteria, the most common being 1) contiguity: all districts must consist of a single connected geographical area; 2) equity: all districts must have roughly the same population or the same number of voters; 3) compactness, which can be measured in several ways (Bozkaya et al., 2003): as much as possible, the districts should have approximately round or square shapes, which helps avoid gerrymandering and often facilitates travel within the district; 4) in some cases it is desirable to create districts that are homogeneous in terms of socio-economic characteristics; 5) major geographical boundaries such as wide rivers or straits should be respected; 6) if a districting plan is created to replace an old one, it may be preferable to maximize the degree of similarity between the two plans. In addition, the process should be politically neutral and not take previous voting results into account.

In political districting, the districts are constructed by agglomerating basic units, such as census tracts, for which geographical, demographic and socio-economic data are available. Contiguity is a hard constraint, but the other constraints can be treated in a multi-objective fashion, for example by associating a function $F(x)$ to any candidate solution x, where $F(x)$ is a weighted sum of measurements associated with each criterion one wishes to consider.

Solving districting problems exactly in the presence of constraints such as contiguity and compactness poses important modeling and algorithmic challenges because of the non-linearities that these constraints induce. Therefore, local search heuristics are most often used instead of exact algorithms (Ricca et al., 2011). Our heuristic (Bozkaya et al., 2003) constructs an initial contiguous solution and applies tabu search to it in the hope of improving it. At each iteration, a basic unit is moved from its district to an adjacent one, or two basic units are exchanged between two adjacent districts. This was the methodology employed by Bozkaya et al. (2011) to redesign the electoral map of Edmonton into 12 districts. The districting plan was approved by the city and was implemented in 2010. This project was awarded the 2010 CORS Practice Prize.

When designing sales territories, it is important to take contiguity and compactness into account, but also to ensure that the workload and possibly the expected revenue of each salesman are equitably distributed. Lei et al. (2012) solved one such problem in a context where some of the customers are deterministic and some others are stochastic, as is often the case in practice. Since the workload depends partly on the length of the route followed to visit the customers associated with any single day, the expected route lengths are approximated by means of the Beardwood et al. (1959) formula which expresses the route length as simple function of the expected number of customers in a district and the area of the district. Lei et al. (2015) later studied a dynamic problem, where customers may enter or leave the system over a planning horizon, and it is desirable not to modify the districts too much over time since the salesmen wish to preserve the rapports they have established with their customers. Lei et al. (2016) have later solved a sales territory districting problem combining stochastic and dynamic customers.

Blais et al. (2003) designed six healthcare delivery districts for a community health clinic in a Montreal borough. They had to account for the indivisibility of some predefined basic units, connectivity of the districts, and workload balance. The latter criterion reflected the amount of work associated with the number and the severity levels of the patients to be visited, and also the travel times. It was assumed that the nurses traveled by bus, which led to rather elongated districts created around bus lines. The problem was solved by tabu search by moving basic units across the boundaries of adjacent districts.

The following two districting applications include a routing component, and neither makes use of two-dimensional basic units. Haugland et al. (2007) considered the problem of grouping customers with random demands into delivery districts, each served by a single vehicle. The problem is formulated as a two-stage stochastic program. In the first stage, the districts are designed, and a tentative vehicle route is constructed in each district. The demands are then revealed and corrective actions such as returns to the depot, are applied. The problem is to compute a first-stage solution so that the expected cost of the realized solution will be minimized. It was observed that such a solution would consist of a single district, so an upper bound on the cost of the first-stage routes was imposed, still leaving the number of districts as a decision variable. Since the model does not work with basic units, but only with points in the plane, the notion of contiguity is elusive and traditional compactness measures have to be adapted to deal with discrete points as opposed to two-dimensional geometric entities.

Butsch et al. (2014) constructed districts in an arc routing context, where the basic units are the edges of a graph. Each edge has a service time as well as a deadheading time. The edges are to be visited in Chinese postman tours, often by employees working on a bicycle or on foot. In such a context, it is preferable to generate solutions with a low deadheading. Two hard constraints are imposed on the districting plan: the districts must define a partition of the edges, and they must be connected. There are also some soft constraints: the districts must be well balanced in terms of workload, the total deadheading time should be small, and the districts must be compact. As in the previous application, traditional compactness measures must be adapted to account for the fact that the basic units are edges as opposed to planar areas. The authors define a local compactness measure which is the total distance of the basic units of a district to its centre, and a global compactness measure is obtained by enclosing each district within the smallest enclosing axis-parallel rectangle that contains all its edges, and computing the total overlap of these rectangles in the districting plan. As in political districting (Bozkaya et al., 2003), a multi-criteria function was minimized by means of a tabu search algorithm.

8 Waste management

Solid waste management has become a top priority everywhere, with the growth in the amount of waste generated and the increasing concern for the protection of the environment (Ghiani et al., 2014a,b; de Souza Melaré et al., 2017). Ghiani et al. (2014a) describe several strategic and tactical issues encountered in solid waste management. The strategic issues include the overall design of the network, the construction of treatment facilities, the location of landfills and incinerators, and the choice of waste transformation technologies. The tactical decisions include districting, the location of recycling bins (Rossit and Nesmachnow, 2022), vehicle fleet composition, collection patterns and scheduling, and the design of garbage collection routes (Ghiani et al., 2014a,c; De Maio et al., 2021), sometimes together with the allocation of vehicle types to the routes (Zbib and Laporte, 2020). Several of these decisions are intertwined and are complicated by the fact that garbage accumulation is stochastic. In some cases, inventory control, made possible by the installation of sensors in public collection bins (Hannan et al., 2015), adds another dimension to the problems.

My interest in waste management stems mostly from a project funded by the Danish Council for Independent Research – Social Sciences, in collaboration with Sanne Wøhlk at Aarhus University. We worked on two main problems: the coordinated capacitated arc routing problem and the transport of skips between recycling centers and treatment facilities. In both cases we had access to real Danish data. With Hani Zbib, I later took part in another project that also made use of Danish data.

In Denmark, like in many other countries, selective collections are organized for general garbage and for several types of materials called fractions. These include glass, plastic, paper, carboard, metal, and organic waste. However, these collections do not all have the same frequency, which tends to range from weekly to four-weekly, and the collection days are not the same for all fractions. This of course complicates the life of citizens who must manage a complicated calendar of collection days. Our research question was to determine how much more expensive it would be to perform the collections for all fractions on the same day of the week. This can only be achieved by designing a heuristic that will determine balanced collection districts, as well as vehicle routes and collection schedules for coordinated collection days or not. Extensive computational experiments were performed on data from five Danish counties, which revealed that coordinated collection days would increase the collection costs by 12.4% and the number of vehicles by 9.1% (Wøhlk and Laporte, 2019). It is worth noting that answering a rather straightforward question necessitated the design and implementation of an advanced algorithmic apparatus, and also required some simplifications to be made since we worked on an undirected graph when in reality there are some one-way streets, and we did not consider complications such as no-turn restrictions. In addition, the graphs on which the problems were solved were quite large, containing as many as 11,656 vertices and 12,691 edges. The vehicle routes were obtained by means of the FastCARP heuristic for the capacitated arc routing problem (Wøhlk and Laporte, 2018).

The second routing problem included in the Danish projects concerned the transport of skips between recycling centers and treatment facilities (Wøhlk and Laporte, 2022). Recycling centers are open-air sites to which citizens drive to offload non-standard garbage such as old tires, wood, textile, furniture, construction debris, electronic equipment, etc. There are 364 such recycling centers in Denmark, each equipped with 20 to 30 skips associated with different garbage types. When a skip becomes full, it is transported by a tractor to a treatment facility where recycling takes place. A tractor can transport up to two full or empty skips between the recycling center and a treatment facility, or between two treatment facilities, and all operations performed by the same tractor must be completed within a given time limit. We studied two versions of the problem: one with fixed returns, where each skip must ultimately return to its origin, and one without fixed returns. The objective is to minimize the sum of the fixed cost associated with the use of the tractors, plus the total routing costs. This problem is akin to the skip collection and delivery problem introduced by De Meulemeester et al. (1997), in which the tractors could carry only one skip at a time. However, allowing the transport of two skips simultaneously significantly increases the difficulty of the problem. We developed a MILP model for the problem and solved it by means of a variable neighbourhood search-inspired heuristic which was tested on 80 real-life data sets from four Danish areas, provided by our industrial partner. These contained up to 185 transportation requests and 17 treatment facilities.

Another project was to design garbage collection routes for heterogeneous compartmented vehicles, as is now the case in several settings (Zbib and Laporte, 2020). The capacity of a garbage compartment depends on the fraction type it contains and on its compression factor. This is an arc routing problem in which three types of decisions must be made: determining the vehicle fleet composition, assigning waste fractions to vehicle compartments, and designing the vehicle routes. The problem was solved by means of a heuristic that decomposes the problem into sequential decision steps. The algorithm was tested on 63 Danish graphs containing up to 6,149 vertices and 3,797 required edges, with three to six fractions to be collected, and four to six vehicle types, having one to four compartments. It was shown that the solution favored combining several fractions in vehicles made up of many compartments. The algorithm worked well for several data types and could easily cope with different distributions of fraction types, especially between cities and rural areas.

Finally, Yu et al. (2020) studied the problem of transporting hazardous waste between several facilities, in a stochastic context, by taking into account the transportation cost and population exposure to risk. The problem was solved by combining sample average approximation and goal programming, and the results were illustrated on medical waste transportation in Wuhan, China.

9 Metro network design

Metro systems constitute an efficient way of transporting large volumes of passengers in major cities. They operate underground, at least in the city cores, consume relatively little energy, help reduce car traffic, and generate very little pollution. There are now more than 210 metro networks worldwide, more than twice the number observed in 1995. See the surveys of Gendreau et al. (1995a) and of Laporte and Mesa (2019).

Metro construction projects are highly complex and fraught with uncertainties, accidents, and cost overruns. They involve several agents, including politicians, city planners, transit agencies, engineers, environmental groups, citizens, and construction companies. The main goal of a metro system is to provide an efficient and cost-effective way of improving mobility for large populations, and several technologies are available (Musso and Vuchic, 1988; Vuchic, 2005; Montoya et al., 2017; Moccia et al., 2020, 2022). To this end, several intertwined decisions need to be made, including the location of the lines (also called alignments) and of the stations, the purchase of rolling stocks, the service frequency, and the fare structure. In addition, it is also important to consider positive and negative externalities, as well as the competition with other modes of transportation when planning a metro network (Gutiérrez-Jarpa et al., 2013, 2017; Canca et al., 2017).

The multiplicity, complexity and interdependency of the decisions involved in the planning of metro systems are such that operational research methods alone cannot solve the problem in its entirety. They can, however, be applied to some well-defined subproblems and provide alternative designs to support the decision-making process. My research in this area has focused on the design of the network.

A question of prime interest is the assessment of basic configurations for metro systems using indicators such as connectivity, travel directness, and number of transfers. Thus, Musso and Vuchic (1988) and Laporte et al. (1997) have shown that cartwheel and triangle designs are preferable to stars (Figure 6) in terms of travel connectivity and directness. Complex networks such as those of London, Paris, Moscow, Tokyo, Shanghai, Seoul, and Barcelona include many connection options that help reduce travel time. However, such networks are rarely built or even fully designed from scratch. They often start as a simple shape, such as a line or a cross, and are later expanded into a more complex configuration. Complex metro systems are also more reliable than simple ones because they provide more alternative routes in case of failure. Laporte et al. (2010, 2011), De Los Santos et al. (2012), D'Lima and Medda (2015) and Mattsson and Jenelius (2015) have analyzed the reliability of rapid transit networks.

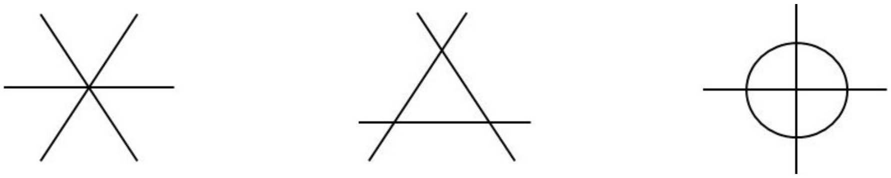
(a) Star **(b) Triangle** **(c) Cartwheel**

Figure 6: Three basic network designs

Well designed metro networks provide a good coverage of the population and of popular locations such as business districts, universities, hospitals, transportation hubs, and major tourist attractions. To measure population coverage, it is common to assume that the proportion of the population of a given point attracted by a station decreases with the distance that separates it from the station. Another common objective is to cover the travel demand, expressed as the number of origin-destination trips, often in a context of competition with an alternative transportation mode, such as the car. Most available network design methodologies simultaneously locate the alignments and the stations, and they do so heuristically. It is convenient and realistic to first define the general shape of the network to be built, such as a single line, a cross, a multi-branch star, a triangle, or a cartwheel. In practical applications, a broad corridor for each alignment can be used by the planners based on their knowledge of the city. For example, a corridor could be a set of parallel streets linking two areas of the city. It is also common to impose constraints on the number of stations and on their spacing, typically between half a kilometer and two kilometers.

Bruno et al. (2002) designed a tabu search heuristic for the location of a maximal population coverage alignment in a graph. This work was extended to the location of a metro network whose individual alignments should belong to prespecified corridors (Bruno and Laporte, 2002). The idea of using predefined corridors has since been applied in most studies. Laporte et al. (1985) were the first to maximize trip coverage for a single alignment. They compared several simple heuristics and concluded to the superiority of a greedy extension constructive heuristic. Gutiérrez-Jarpa et al. (2013) considered the problem of locating a maximal trip capturing network. They restricted the network to belong to a configuration belonging to predefined corridors and solved the problem exactly by CPLEX. Laporte and Pascoal (2015) considered two criteria: population coverage and construction costs, again using corridors. Non-dominated alignments were generated within each corridor and then assembled to form a desired configuration by solving a mathematical model. Gutiérrez-Jarpa et al. (2017) extended the work of Gutiérrez-Jarpa et al. (2013) by considering this time three criteria: the minimization of construction costs, the maximization of travel time savings with respect to the competing mode, and patronage maximization. They then conducted post-optimization analyses to assess their solutions in terms of effectiveness, efficiency, and equity. Gutiérrez-Jarpa et al. (2018) designed a matheuristic to solve a demand-capturing network, where the captured demand depends on the ratio of the travel time by car and of the travel time using the metro. Their results were illustrated on data from the city of Concepcíon in Chile.

When a metro network is designed, the alignments and the stations are located simultaneously. However, it is possible to fine-tune the solution by post-optimizing the station locations. To this end, Laporte et al. (2002) proposed a methodology based on the use of catchment areas and census tracts. In addition, the exact trajectories of the underground tunnels can be optimized based on geological or construction cost considerations, without affecting the network topology. An example is the segment of the Seville metro connecting the bus station (Plaza de Armas) and the train station (Santa Justa) that had to deviate from a straight line to avoid being too close to some historical monuments in the old part of the city (Laporte et al., 2009). The problem was solved by iteratively computing shortest paths on a Voronoi diagram.

Finally, Canca et al. (2019) and Canca and Laporte (2022) solved the problem of optimally scheduling the main phases of a metro construction project. Very often, metro systems are built over a long planning horizon which may encompass several decades. To schedule the main construction phases, the network can be broken into segments linking key stations for example, and these segments will be built separately. Each segment generates a construction cost and a projected revenue when completed. The problem is to determine an optimal schedule for the construction of the segments, considering the discounted costs and revenues over the planning horizon. These two papers consider the deterministic and the stochastic cases, respectively.

10 Green transportation

The growing concerns about climate change and the environment have stimulated research into ways of reducing energy consumption and pollution in all modes of transportation. My research in this area has focused on speed control and on electric vehicles for goods distribution.

In maritime transportation, energy consumption is a convex function of speed that is close to a cubic function (Psaraftis and Kontovas, 2013). Fagerholt et al. (2010) have considered the problem of optimizing speed on a linear ship route consisting of several segments connecting n ports of visit. A time window is imposed on the arrival time at each port, and the problem is to optimize speed on each segment in order to respect the time window constraints. Hvattum et al. (2013) have devised an

$O(n^2)$ exact algorithm for this problem, applicable to any convex energy consumption function. Other applications of speed optimization in the maritime industry include Norstad et al. (2011) and Norlund et al. (2015). Zhuge et al. (2020, 2021) describe programs aimed at reducing vessel speed, while Wang et al. (2022) investigate the use of subsidies to promote the adoption of shore power in ship berthing operations.

Bektaş and Laporte (2011) introduced the pollution-routing problem which consists of computing vehicle routes generating the least amount of pollution in the VRP. Their model is partly based on the vehicle energy consumption equations of Barth et al. (2005) and of Barth and Boriboonsomsin (2009). While several independent variables explain energy consumption, three key factors emerge: distance, the vehicle load, and speed. If the speed exceeds 40 km/h, then energy consumption can be approximated by the function energy $= A\,(\text{load} \times \text{distance}) + B\,(\text{speed}^2 \times \text{distance})$, where A and B are parameters, which implies that travel direction has an impact on energy consumption since it affects the vehicle load. Including time windows and travel costs adds further complications. For speeds in excess of 40 km/h, traveling faster increases energy consumption but reduces travel cost if the drivers are paid on an hourly basis, which yields a bicriteria problem (Demir et al., 2014). The pollution-routing problem can be formulated as a non-linear integer program which can be linearized by discretizing speed (Bektaş and Laporte, 2011). It was also solved heuristically by Demir et al. (2012). Numerous extensions to the pollution-routing problem have been proposed, namely to include time-dependent speeds (Franceschetti et al., 2013, 2017, 2018), fleet size and mix (Koç et al., 2014), and depot location (Koç et al., 2016). In practice, speed optimization is impossible to achieve in congested road networks, but it can prove a viable option in the case of autonomous trucks (Nasri et al., 2018).

The use of electric vehicles for goods transportation is becoming increasingly popular and has given rise to a rich research area in recent years (Pelletier et al., 2016; Schiffer, 2017; Sanguesa et al., 2021). Electric vehicles typically operate with lithium-ion batteries, which has consequences on models and algorithms. First, these batteries take a long time to recharge, and the number of available charging stations is sometimes limited. In addition, the charging process is a concave function of time (Montoya et al., 2017), which means that the recharging level is often a decision variable in mathematical models. Batteries are expensive and their lifetime degradation depends on their charging and discharging behaviour and frequency, as well as how they are maintained (Pelletier et al., 2017). There exist several models and algorithms that integrate routing and charging decisions (for example those of Bruglieri et al. (2017, 2019); Koç et al. (2019); Macrina et al. (2019a,b); Froger et al. (2019, 2022)). Some authors have considered congestion at charging stations (Keskin et al., 2019, 2021), charge scheduling when vehicles recharge at the depot overnight (Pelletier et al., 2018), and the need to design robust vehicle routes in the face of energy consumption uncertainty (Pelletier et al., 2019a). Most studies in the field of electric vehicles are concerned with the operational aspects (routing and charging decisions). Some exceptions are the work of Pelletier et al. (2019b) which considers the strategic long-term problem of transitioning from a conventional vehicle fleet to an electric fleet, be it for buses or trucks, and the paper of Schiffer et al. (2021) which develops an ownership strategy for electric vehicles in the context of a mid-haul logistics network.

11 Humanitarian logistics

Humanitarian logistics is another important and fast-growing research topic. My involvement in this area is relatively recent and was prompted by cooperation with my colleague and former Ph.D. student Marie-Ève Rancourt. All our contributions contain an important location or network design component, as is often the case in humanitarian logistics (Kara and Rancourt, 2019).

The first project, which was part of Rancourt's doctoral thesis, was to develop a methodology for the design of a food aid distribution network in the Garissa region of Kenya (Rancourt et al., 2015). This network contains main warehouses (MWs) used as transshipment points and storage facilities, and distribution centers (DCs). The aim is to locate several DCs that are accessed by the beneficiaries to get hold of their food. The transportation cost between the MWs and the DCs is borne by the World Food Program, whereas the beneficiaries cover their own costs. The model that was developed imposes a limit on the walking distance of the beneficiaries to their closest DC. It is an uncapacitated plant location model whose objective is made up of the distribution costs from the MWs and the DCs, the walking costs of the beneficiaries, and the hand-out costs at the DCs. This model is interesting in that it combines some features of coverage models and of distance minimization models. The fact that a coverage constraint is applied does not mean that the beneficiaries located outside the coverage radius are not served, but only that they are not considered in the plant location model in order not to bias the solution toward excentric beneficiaries.

A second project in which I was involved was also related to Sub-Saharan East Africa. It consisted of comparing two supply chain networks (Dufour et al., 2018) used by the United Nations Humanitarian Response Depot (UNHRD) to distribute products to end-users in parts of Africa. The first network, which was the current one, consisted of suppliers, three depots located in Brindisi, Dubai and Accra, and the end-users. The second network would also contain a regional distribution center (RDC) located in Kampala, Uganda, between the depots and the end-users, but the depots would still be used to ship some goods directly to the end-users. The RDC is a very simple physical structure consisting of 10 twenty-foot containers located in a facility already owned by the UNHRD and hence entails no extra cost. The question was to determine the benefit of using an RDC and of determining which products, among the 15 that were currently distributed, should be stored in the 10 containers. We conducted a statistical analysis and generated 5,000 demand scenarios, and for each scenario two network design problems were solved exactly by CPLEX. The analysis led us to conclude that locating an RDC in Kampala would reduce the average distribution cost by 21%. It also allowed us to identify the six products that should be stored in the RDC. The proposed solution was implemented by the UNHRD and the project won the CORS Practice Prize in 2016.

Every year, several subsets of Caribbean countries are affected by hurricanes. Risk pooling benefits can be obtained by adopting a collaborative prepositioning strategy for emergency supply inventories. Balçık et al. (2019) cooperated with the Caribbean Disaster and Emergency Management Agency to develop a prepositioning and cost sharing plan. They developed a stochastic programming model aimed at determining the locations and the quantities of supplies, as well as the country investments. The premiums are related to the expected value and to the standard deviation of the countries' demands. Extensive test results confirmed that significant inventory reductions are achievable through collaborative prepositioning. In a subsequent study, Rodríguez-Pereira et al. (2021) considered several cost allocation methods based on the countries' risk levels and ability to pay. They compared several schemes such as the use of the Shapley value, the alternative cost allocation method, and the equal profit method, and they developed a novel insurance-based policy based on the solution of a MILP. These methods were compared by means of several equity metrics, and the superiority of the newly proposed insurance-based policy was demonstrated with respect to equity and computational efficiency.

Refugee crises constitute another major humanitarian problem in several countries. Arslan et al. (2021) studied the problem faced by Red Crescent of providing basic services such as education, healthcare and safety to refugees living in camps in Southeast Turkey. The problem is to jointly locate the camps and plan the transportation between the service providers and the camps, which can be modeled and solved as a location-routing problem. The authors developed a branch-and-price-and-cut algorithm capable of optimally solving real-world instances involving up to 244 nodes.

Finally, Laporte et al. (2022) modeled and solved the problem of designing a drinking water distribution network in a remote area, with application to Nepal. This study was motivated by the need to restore the network that was destroyed by the two 2015 earthquakes in the districts of Gorkha and Dolakha. The problem decomposes naturally into two subproblems which must be solved hierarchically because the first is deemed far more important than the second. The first subproblem, which is to locate water taps subject to accessibility constraints, is solved exactly as a fixed-charge facility location problem. The second is to connect these water taps to water sources by means of a

Steiner forest, taking into account the fact that the water distribution system is gravity-fed and must respect some technical constraints. This subproblem is solved heuristically by simulated annealing. Tests performed on two village communities of the Dolakha district confirmed the effectiveness of the proposed methodology. In 2022 this work received the SEIO-BBVA prize awarded to the best paper in operational research by the Spanish Society for Statistics and Operational Research, and the BBVA banking group foundation.

12 Conclusions

In 1971, while I was completing a bachelor's degree in mathematics at McGill University, I made the decision to study operational research because I was more interested in applications than in theory. I then completed my master's degree in operational research at Lancaster University and my Ph.D. at the London School of Economics. I never regretted my choice: operational research is everything I expected and more. This field is indeed very rich in applications of all kinds, as I just illustrated. It is multidisciplinary and its methodological scope is also broad since one can conduct projects ranging from research based on applied mathematics or computer science, to consultancy work. It combines logic, rigor, creativity, and practical sense. Operational research clearly applies to a wide range of topics, starting from military and industrial applications from which it originates, to contemporary concerns such as waste management, green transportation and humanitarian logistics. It even provides room for the study of playful problems like dartboard design (Eiselt and Laporte, 1991) and Sudoku puzzles (Coelho and Laporte, 2014). I always found my formal mathematical training very useful in conceptualizing problems, in formulating models, and in designing algorithms.

The 10 research areas I have described in this paper illustrate the breadth of my interests and of what operational research has to offer, but I had to leave out several other interesting applications I have also studied. It is certainly stimulating to work on a wide range of problems, but it is equally important to acquire some methodological depth. Quite often I have revisited the same problem many times over a span of several years because there is always something to improve, new techniques come into the fore, and over time one establishes new connections between different problems and methodologies. The best advice I can give to any operational researcher is to work on problems one enjoys, develop scientific collaborations, and integrate research into teaching.

Over the past 50 years, we have witnessed significant progress in the theory and practice of operational research. Some of the most noticeable advances relate to the development of powerful MILP solvers such as CPLEX and Gurobi, while others apply to the field of metaheuristics. These advances, combined with the fast growth of computer and information technologies, mean that large classes of hard problems that were previously intractable can now be solved relatively quickly for realistic sizes with a high degree of accuracy. Further developments are to be expected through the integration of machine learning techniques in existing algorithmic frameworks, namely in those that pertain to combinatorial optimization. In this respect, the recent papers of Bengio et al. (2021) and of Karimi-Mamaghan et al. (2022) provide several promising research directions.

References

Accorsi, L., Vigo, D., 2021. A fast and scalable heuristic for the solution of large-scale capacitated vehicle routing problems. Transportation Science 55, 832–856.

Adelson, R.M., Norman, J.M., Laporte, G., 1976. A dynamic programming formulation with diverse applications. Operational Research Quarterly 27, 119–121.

Arslan, O., Çulhan Kumcu, G., Kara, B.Y., Laporte, G., 2021. The location and location-routing problem for the refugee camp network design. Transportation Research Part B: Methodological 143, 201–220.

Atkins, J.E., Boman, E.G., Hendrickson, B., 1998. A spectral algorithm for seriation and the consecutive ones problem. SIAM Journal on Computing 28, 297–310.

Balakrishnan, N., Wong, R.T., 1990. A network model for the rotating workforce scheduling problem. Networks 20, 25–42.

Balçık, B., Silvestri, S., Rancourt, M.-È., Laporte, G., 2019. Collaborative prepositioning network design for regional disaster response. Production and Operations Management 28, 2431–2455.

Barth, M., Boriboonsomsin, K., 2009. Energy and emissions impacts of a freeway-based dynamic eco-driving system. Transportation Research Part D: Transport and Environment 14, 400–410.

Barth, M., Younglove, T., Scora, G., 2005. Development of a heavy-duty diesel modal emissions and fuel consumption model. Technical report UCB-ITS-PRR-2005-1. California PATH Program, lnstitute of Transportation Studies, University of California at Berkeley.

Beardwood, J., Halton, J.H., Hammersley, J.M., 1959. The shortest path through many points. Mathematical Proceedings of the Cambridge Philosophical Society 55, 299–327.

Bektaş, T., Laporte, G., 2011. The pollution-routing problem. Transportation Research Part B: Methodological 45, 1232–1250.

Benders, J.F., 1962. Partitioning procedures for solving mixed-variables programing problems. Numerische Mathematik 4, 238–252.

Bengio, Y., Lodi, A., Prouvost, A., 2021. Machine learning for combinatorial optimization: A methodological tour d'horizon. European Journal of Operational Research 290, 405–421.

Bertsimas, D.J., 1992. A vehicle routing problem with stochastic demand. Operations Research 40, 574–585.

Bertsimas, D.J., Jaillet, P., Odoni, A.R., 1990. A priori optimization. Operations Research 38, 1019–1033.

Blais, M., Lapierre, S.D., Laporte, G., 2003. Solving a home-care districting problem in an urban setting. Journal of the Operational Research Society 54, 1141–1147.

Booth, K.S., 1975. PQ-Tree Algorithms. University of California, Berkeley.

Bozkaya, B., Erkut, E., Haight, D., Laporte, G., 2011. Designing new electoral districts for the city of Edmonton. Interfaces 59, 534–547.

Bozkaya, B., Erkut, E., Laporte, G., 2003. A tabu search heuristic and adaptive memory procedure for political districting. European Journal of Operational Research 144, 12–26.

Brainerd, G.W., 1951. The place of chronological ordering in archaeological analysis. American Antiquity 16, 301–313.

Brélaz, D., 1979. New methods to color the vertices of a graph. Communications of the ACM 22, 251–256.

Bruglieri, M., Mancini, S., Pezzella, F., Pisacane, O., 2019. A path-based solution approach for the green vehicle routing problem. Computers & Operations Research 103, 109–122.

Bruglieri, M., Mancini, S., Pezzella, F., Pisacane, O., Suraci, S., 2017. A three-phase matheuristic for the time-effective electric vehicle routing problem with partial recharges. Electronic Notes in Discrete Mathematics 58, 95–102.

Bruno, G., Gendreau, M., Laporte, G., 2002. A heuristic for the location of a rapid transit line. Computers & Operations Research 29, 1–12.

Bruno, G., Laporte, G., 2002. An interactive decision support system for the design of rapid public transit networks. INFOR: Information Systems and Operational Research 40, 111–118.

Butsch, A., Kalcsics, J., Laporte, G., 2014. Districting for arc routing. INFORMS Journal on Computing 26, 809–824.

Canca, D., De Los Santos, A., Laporte, G., Mesa, J.A., 2017. An adaptive neighborhood search metaheuristic for the integrated railway rapid transit network design and line planning problem. Computers & Operations Research 78, 1–14.

Canca, D., De Los Santos, A., Laporte, G., Mesa, J.A., 2019. The railway rapid transit network construction scheduling problem. Computers & Industrial Engineering 138, 106075.

Canca, D., Laporte, G., 2022. Solving a real-size stochastic railway rapid transit network construction scheduling problem. Computers & Operations Research 138, 105600.

Carøe, C.C., Tind, J., 1998. L-shaped decomposition of two-stage stochastic programs with integer recourse. Mathematical Programming 83, 451–464.

Carter, M.W., 1986. A survey of practical applications of examination timetabling algorithms. Operations Research 34, 193–202.

Carter, M.W., Laporte, G., 1995. Recent developments in practical examination timetabling, in: Burke, E.K., Ross, P. (Eds.), PATAT 1995: Practice and Theory of Automated Timetabling, Springer, Heidelberg, Berlin. pp. 1–21.

Carter, M.W., Laporte, G., Chinneck, J.W., 1994. A general examination scheduling system. Interfaces 24, 109–120.

Carter, M.W., Laporte, G., Lee, S.Y., 1996. Examination timetabling: Algorithmic strategies and applications. Journal of the Operational Research Society 47, 373–383.

Christofides, N., 1976. The vehicle routing problem. Revue française d'automatique, d'informatique et de recherche opérationnelle. Recherche opérationnelle 10, 55–70.

Chvátal, V., 1973. Edmonds polytopes and a hierarchy of combinatorial problems. Discrete Mathematics 4, 305–337.

Coelho, L.C., Laporte, G., 2014. A comparison of several enumerative algorithms for Sudoku. Journal of the Operational Research Society 65, 1602–1610.

Dantzig, G.B., Fulkerson, D.R., Johnson, S.M., 1954. Solution of a large-scale traveling-salesman problem. Journal of the Operations Research Society of America 2, 393–410.

Dantzig, G.B., Ramser, J.H., 1959. The truck dispatching problem. Management Science 6, 80–91.

De Los Santos, A., Laporte, G., Mesa, J.A., Perea, F., 2012. Evaluating passenger robustness in a rail transit network. Transportation Research Part C: Emerging Technologies 20, 34–46.

De Maio, A., Laganà, D., Musmanno, R., Vocaturo, F., 2021. Arc routing under uncertainty: Introduction and literature review. Computers & Operations Research 135, 105442.

De Meulemeester, L., Laporte, G., Louveaux, F.V., Semet, F., 1997. Optimal sequencing of skip collections and deliveries. Journal of the Operational Research Society 48, 57–64.

Demir, E., Bektaş, T., Laporte, G., 2012. An adaptive large neighborhood search heuristic for the pollution-routing problem. European Journal of Operational Research 223, 346–359.

Demir, E., Bektaş, T., Laporte, G., 2014. The bi-objective pollution-routing problem. European Journal of Operational Research 232, 464–478.

de Souza Melaré, A.V., Montenegro González, S., Faceli, K., Casadei, V., 2017. Technologies and decision support systems to aid solid-waste management: A systematic review. Waste Management 59, 567–584.

D'Lima, M., Medda, F., 2015. A new measure of resilience: An application to the London underground. Transportation Research Part A: Policy and Practice 81, 35–46.

Doran, J.E., Powell, S., 1972. Solving a combinatorial problem encountered in archaeology, in: Some Research Applications of the Computer. Atlas Computer Laboratory, Harwell, Oxfordshire, pp. 47–52.

Dror, M., Laporte, G., Louveaux, F.V., 1993. Vehicle routing with stochastic demands and restricted failures. Zeitschrift für Operations Research 37, 273–283.

Dror, M., Laporte, G., Trudeau, P., 1989. Vehicle routing with stochastic demands: Properties and solution frameworks. Transportation Science 23, 166–176.

Dufour, E., Laporte, G., Paquette, J., Rancourt, M.-È., 2018. Logistics service network design for humanitarian response in East Africa. Omega: The International Journal of Management Science 74, 1–14.

Eiselt, H.A., Laporte, G., 1991. A combinatorial optimization problem arising in dartboard design. Journal of the Operational Research Society 42, 113–118.

Fagerholt, K., Laporte, G., Norstad, I., 2010. Reducing fuel emissions by optimizing speed on shipping routes. Journal of the Operational Research Society 61, 523–529.

Franceschetti, A., Demir, E., Honhon, D., Van Woensel, T., Laporte, G., Stobbe, M., 2018. A metaheuristic for the time-dependent pollution-routing problem. European Journal of Operational Research 259, 972–991.

Franceschetti, A., Honhon, D., Laporte, G., Van Woensel, T., 2017. A shortest-path algorithm for the departure time and speed optimization problem. Transportation Science 52, 756–768.

Franceschetti, A., Honhon, D., Van Woensel, T., Bektaş, T., Laporte, G., 2013. The time-dependent pollution-routing problem. Transportation Research Part B: Methodological 56, 265–293.

Froger, A., Jabali, O., Mendoza, J.E., Laporte, G., 2022. The electric vehicle routing problem with capacitated charging stations. Transportation Science 56, 460–482.

Froger, A., Mendoza, J.E., Jabali, O., Laporte, G., 2019. Improved formulations and algorithmic components for the electric vehicle routing problem with nonlinear charging functions. Computers & Operations Research 104, 256–294.

Fulkerson, D.R., Gross, O., 1965. Incidence matrices and interval graphs. Pacific Journal of Mathematics 15, 835–855.

Garey, M.R., Johnson, D.S., 1979. Computers and Intractability: A Guide to the Theory of NP-Completeness. W.H. Freeman & Co., San Francisco.

Gendreau, M., Hertz, A., Laporte, G., 1994. A generalized insertion algorithm for the seriation problem. Mathematical and Computer Modelling 19, 53–59.

Gendreau, M., Jabali, O., Rei, W., 2016. 50th anniversary invited article—Future research directions in stochastic vehicle routing. Transportation Science 50, 1163–1173.

Gendreau, M., Laporte, G., Mesa, J.A., 1995a. Locating rapid transit lines. Journal of Advanced Transportation 29, 145–162.

Gendreau, M., Laporte, G., Séguin, R., 1995b. The vehicle routing problem with stochastic demands and customers. Transportation Science 29, 143–155.

Gendreau, M., Laporte, G., Séguin, R., 1996. A tabu search heuristic for the vehicle routing problem with stochastic demands and customers. Operations Research 44, 469–477.

Ghiani, G., Laganà, D., Manni, E., Musmanno, R., Vigo, D., 2014a. Operations research in solid waste management: A survey of strategic and tactical issues. Computers & Operations Research 44, 22–32.

Ghiani, G., Manni, A., Manni, E., Toraldo, M., 2014b. The impact of an efficient collection sites location on the zoning phase in municipal solid waste management. Waste Management 34, 1949–1956.

Ghiani, G., Mourão, C., Pinto, L., Vigo, D., 2014c. Routing in waste collection, in: Corberán, Á., Laporte, G. (Eds.), Arc Routing. Problems, Methods and Applications. MOS-SIAM Series on Optimization, Philadelphia, pp. 351–370.

Gutiérrez-Jarpa, G., Laporte, G., Marianov, V., 2018. Rapid transit network design for optimal cost and origin-destination demand capture. Computers & Operations Research 89, 58–67.

Gutiérrez-Jarpa, G., Laporte, G., Marianov, V., Moccia, L., 2017. Multi-objective rapid transit network design with modal competition: The case of Concepción, Chile. Computers & Operations Research 78, 27–43.

Gutiérrez-Jarpa, G., Obreque, C., Laporte, G., Marianov, V., 2013. Rapid transit network design for optimal cost and origin-destination demand capture. Computers & Operations Research 40, 3000–3009.

Hannan, M.A., Al Mamum, M.A., Basri, H., Begum, R., 2015. A review on technologies and their usage in solid waste monitoring and management systems: Issues and challenges. Waste Management 43, 509–523.

Haugland, D., Ho, S.C., Laporte, G., 2007. Designing delivery districts for the vehicle routing problem with stochastic demands. European Journal of Operational Research 180, 997–1010.

Hodson, F.R., 1968. The La Tène Cemetery at Münsingen-Rain. Stämpfli, Berne.

Hoogendoorn , Y.N., Spliet, R., 2023. An improved integer L-shaped method for the vehicle routing problem with stochastic demands. INFORMS Journal on Computing 35, 423–439.

Hvattum, L.M., Norstad, I., Fagerholt, K., Laporte, G., 2013. Analysis of an exact algorithm for the vessel speed optimization problem. Networks 62, 132–135.

Jabali, O., Rei, W., Gendreau, M., Laporte, G., 2014. Partial-route inequalities for the multi-vehicle routing problem with stochastic demands. Discrete Applied Mathematics 177, 121–136.

Jaillet, P., 1985. Probabilistic Traveling Salesman Problems. Ph.D. thesis. Massachusetts Institute of Technology.

Kalcsics, J., Ríos-Mercado, R.Z., 2019. Districting problems, in: Laporte, G., Nickel, S., Saldanha da Gama, F. (Eds.), Location Science (Second Edition). Springer, Cham, pp. 705–743.

Kara, B.Y., Rancourt, M.-È., 2019. Location problems in humanitarian supply chains, in: Laporte, G., Nickel, S., Saldanha da Gama, F. (Eds.), Location Science (Second Edition). Springer, Cham, pp. 611–629.

Karimi-Mamaghan, M., Mohammadi, M., Meyer, P., Karimi-Mamaghan, A.M., Talbi, E.-G., 2022. Machine learning at the service of meta-heuristics for solving combinatorial optimization problems: A state-of-the-art. European Journal of Operational Research 296, 393–422.

Kendall, D.G., 1969a. Incidence matrices, interval graphs and seriation in archeology. Pacific Journal of Mathematics 28, 565–570.

Kendall, D.G., 1969b. A mathematical approach to seriation. Philosophical Transactions of the Royal Society 269, 125–135.

Keskin, M., Çatay, B., Laporte, G., 2021. A simulation-based heuristic for the electric vehicle routing problem with time windows and stochastic waiting times at recharging stations. Computers & Operations Research 125, 105060.

Keskin, M., Laporte, G., Çatay, B., 2019. Electric vehicle routing problem with time-dependent waiting times at recharging stations. Computers & Operations Research 107, 77–94.

Knauth, P., 1996. Designing better shift systems. Applied Ergonomics 27, 39–44.

Koç, Ç., Bektaş, T., Jabali, O., Laporte, G., 2014. The fleet size and mix pollution-routing problem. Transportation Research Part B: Methodological 70, 239–254.

Koç, Ç., Bektaş, T., Jabali, O., Laporte, G., 2016. The impact of depot location, fleet composition and routing on emissions in city logistics. Transportation Research Part B: Methodological 84, 81–102.

Koç, Ç., Mendoza, J.E., Jabali, O., Laporte, G., 2019. The electric vehicle routing problem with shared charging stations. International Transactions in Operational Research 26, 1211–1243.

Lambert, V., Laporte, G., Louveaux, F.V., 1993. Designing collections routes through bank branches. Computers & Operations Research 20, 783–791.

Laporte, G., 1975. Permutation Programming: Problems, Methods and Applications. Ph.D. thesis. University of London.

Laporte, G., 1976. A comparison of two norms in archaeological seriation. Journal of Archaeological Science 3, 249–255.

Laporte, G., 1987. Solving a family of permutation problems on 0-1 matrices. Revue française d'automatique, informatique, recherche opérationnelle. Recherche opérationnelle 21, 65–85.

Laporte, G., 1999. The art and science of designing rotating schedules. Journal of the Operational Research Society 50, 1011–1017.

Laporte, G., 2021. Some contributions of Ailsa H. Land to the study of the traveling salesman problem. EURO Journal on Computational Optimization 9, 100018.

Laporte, G., Desroches, S., 1984. Examination timetabling by computer. Computers & Operations Research 11, 351–360.

Laporte, G., Louveaux, F.V., 1993. The integer L-shaped method for stochastic integer programs with complete recourse. Operations Research Letters 13, 133–142.

Laporte, G., Louveaux, F.V., Mercure, H., 1989. Models and exact solutions for a class of stochastic location-routing problems. European Journal of Operational Research 39, 71–78.

Laporte, G., Louveaux, F.V., Mercure, H., 1992. The vehicle routing problem with stochastic travel times. Transportation Science 26, 161–170.

Laporte, G., Louveaux, F.V., Mercure, H., 1994. A priori optimization of the probabilistic traveling salesman problem. Operations Research 42, 543–549.

Laporte, G., Louveaux, F.V., Van hamme, L., 2002. An integer L-shaped algorithm for the capacitated vehicle routing problem with stochastic demands. Operations Research 50, 415–123.

Laporte, G., Marín, Á., Mesa, J.A., Perea, F., 2011. Designing robust transit networks with alternative routes. Journal of Advanced Transportation 45, 54–65.

Laporte, G., Mesa, J.A., 2019. The design of rapid transit networks, in: G. Laporte, Nickel, S., Saldanha da Gama, F. (Eds.), Location Science (Second Edition). Springer, Cham, pp. 687–703.

Laporte, G., Mesa, J.A., Ortega, F.A., 1997. Assessing the efficiency of rapid transit configurations. TOP 5, 95–104.

Laporte, G., Mesa, J.A., Ortega, F.A., Pozo, M.A., 2009. Locating a metro line in a historical city centre: Application to Sevilla. Journal of the Operational Research Society 60, 1462–1466.

Laporte, G., Mesa, J.A., Perea, F., 2010. A game theoretic framework for the robust railway network design problem. Transportation Research Part B: Methodological 44, 447–459.

Laporte, G., Nobert, Y., 1983. A branch and bound algorithm for the capacitated vehicle routing problem. Operations Research Spektrum 5, 77–85.

Laporte, G., Nobert, Y., 1984. Comb inequalities for the vehicle routing problem. Methods of Operations Research 51, 271–276.

Laporte, G., Nobert, Y., Biron, J., 1980. Rotating schedules. European Journal of Operational Research 4, 24–30.

Laporte, G., Nobert, Y., Desrochers, M., 1985. Optimal routing under capacity and distance restrictions. Operations Research 33, 1050–1073.

Laporte, G., Pascoal, M.M.B., 2015. Path based algorithms for metro networks design. Computers & Operations Research 62, 78–94.

Laporte, G., Pesant, G., 2004. A general multi-shift scheduling system. Journal of the Operational Research Society 55, 1208–1217.

Laporte, G., Rancourt, M.-È., Rodríguez-Pereira, J., Silvestri, S., 2022. Optimizing access to drinking water in remote areas. Application to Nepal. Computers & Operations Research 140, 105669.

Laporte, G., Ropke, S., Vidal, T., 2014. Heuristics for the vehicle routing problem, in: Toth, P., Vigo, D. (Eds.), Vehicle Routing. Problems, Methods and Applications. MOS-SIAM Series on Optimization, Philadelphia, pp. 87–116.

Laporte, G., Taillefer, S., 1987. An efficient interchange procedure for the archaeological seriation problem. Journal of Archaeological Science 14, 283–289.

Lei, H., Laporte, G., Guo, B., 2011. The capacitated vehicle routing problem with stochastic demands and time windows. Computers & Operations Research 38, 1775–1783.

Lei, H., Laporte, G., Guo, B., 2012. Districting for routing with stochastic customers. EURO Journal on Transportation and Logistics 1, 67–85.

Lei, H., Laporte, G., Liu, Y., Zhang, T., 2015. Dynamic design of sales territories. Computers & Operations Research 56, 84–92.

Lei, H., Wang, R., Laporte, G., 2016. Solving a multi-objective dynamic stochastic districting routing problem with a co-evolutionary algorithm. Computers & Operations Research 67, 12–24.

Liiv, I., 2010. Seriation and matrix reordering methods: An historical overview. Statistical Analysis and Data Mining: The ASA Data Science Journal 3, 70–91.

Macrina, G., Di Puglia Pugliese, L., Guerriero, F., Laporte, G., 2019a. The green mixed fleet vehicle routing problem with partial battery recharging and time windows. Computers & Operations Research 101, 183–199.

Macrina, G., Laporte, G., Guerriero, F., Di Puglia Pugliese, L., 2019b. An energy-efficient green-vehicle routing problem with mixed vehicle fleet, partial battery recharging and time windows. European Journal of Operational Research 276, 971–982.

Mattsson, L.-G., Jenelius, E., 2015. Vulnerability and resilience of transport systems – a discussion of recent research. Transportation Research Part A: Policy and Practice 81, 16–34.

McCollum, B., McMullan, P., Parkes, A.J., Burke, E.K., Qu, R., 2012. A new model for automated examination timetabling. Annals of Operations Research 194, 291–315.

Miliotis, P., 1976. Integer programming approaches to the travelling salesman problem. Mathematical Programming 10, 367–378.

Miliotis, P., 1978. Using cutting planes to solve the symmetric travelling salesman problem. Mathematical Programming 15, 1436–4646.

Moccia, L., Allen, D.W., Laporte, G., 2020. A spatially disaggregated model for the technology selection and design of a transit line. Public Transport 12, 647–691.

Moccia, L., Allen, D.W., Laporte, G., Spinosa, A., 2022. Mode boundaries of automated metro and semi-rapid rail in urban transit. Public Transport 14, 739–802.

Möhring, R.H., 1990. Graph problems related to gate matrix layout and PLA folding, in: Tinhofer, G., Mayr, E., Noltemeier, H., Syslo, M. (Eds.), Computational Graph Theory. Springer-Verlag, Vienna, pp. 17–51.

Montoya, A., Guéret, C., Mendoza, J.E., Villegas, J.G., 2017. The electric vehicle routing problem with nonlinear charging function. Transportation Research Part B: Methodological 103, 87–110.

Musliu, N., Gärtner, J., Slany, W., 2002. Efficient generation of rotating workforce schedules. Discrete Applied Mathematics 118, 85–98.

Musso, A., Vuchic, V.R., 1988. Characteristics of metro networks and methodology for their evaluation. Transportation Research Record 1162, 22–33.

Naddef, D., Rinaldi, G., 2002. Branch-and-cut algorithms for the capacitated VRP, in: Toth, P., Vigo, D. (Eds.), The Vehicle Routing Problem. SIAM Monographs on Discrete Mathematics and Applications, Philadelphia, pp. 53–84.

Nasri, M.I., Bektaş, T., Laporte, G., 2018. Route and speed optimization for autonomous trucks. Computers & Operations Research 100, 89–101.

Norlund, E., Gribkovskaia, I., Laporte, G., 2015. Supply vessel planning under cost, environment and robustness considerations. Omega: The International Journal of Management Science 57B, 271–281.

Norstad, I., Fagerholt, K., Laporte, G., 2011. Tramp ship routing and scheduling with speed optimization. Transportation Research Part C: Emerging Technologies 19, 853–865.

Oyola, J., Arntzen, H., Woodruff, D.L., 2017. The stochastic vehicle routing problem, a literature review, part II: Solution methods. EURO Journal on Transportation and Logistics 6, 349–388.

Oyola, J., Arntzen, H., Woodruff, D.L., 2018. The stochastic vehicle routing problem, a literature review, part I: Models. EURO Journal on Transportation and Logistics 7, 193–221.

Papadimitriou, C.H., 1976. The NP-completeness of the bandwidth minimization problem. Computing 16, 263–270.

Pecin, D., Pessoa, A., Poggi, M., Uchoa, E., 2014. Improved branch-cut-and-price for capacitated vehicle routing, in: Lee, J., Vygen, J. (Eds.), Integer Programming and Combinatorial Optimization. Springer, Cham, pp. 393–403.

Pecin, D., Pessoa, A., Poggi, M., Uchoa, E., 2017. Improved branch-cut-and-price for capacitated vehicle routing. Mathematical Programming Computation 9, 61–100.

Pelletier, S., Jabali, O., Laporte, G., 2016. 50^{th} anniversary invited article—Goods distribution with electric vehicles: Review and research perspectives. Transportation Science 50, 3–22.

Pelletier, S., Jabali, O., Laporte, G., 2018. Charge scheduling for electric freight vehicles. Transportation Research Part B: Methodological 115, 246–269.

Pelletier, S., Jabali, O., Laporte, G., 2019a. The electric vehicle routing problem with energy consumption uncertainty. Transportation Research Part B: Methodological 126, 225–255.

Pelletier, S., Jabali, O., Laporte, G., Veneroni, M., 2017. Battery degradation and behaviour for electric vehicles: Review and numerical analyses of several models. Transportation Research Part B: Methodological 103, 158–187.

Pelletier, S., Jabali, O., Mendoza, J.E., Laporte, G., 2019b. The electric bus fleet transition problem. Transportation Research Part C: Emerging Technologies 109, 174–193.

Poggi, M., Uchoa, E., 2014. New exact algorithms for the capacitated vehicle routing problem, in: Toth, P., Vigo, D. (Eds.), Vehicle Routing. Problems, Methods, and Applications. MOS-SIAM Series on Optimization, Philadelphia, pp. 59–86.

Psaraftis, H.N., Kontovas, C.A., 2013. Speed models for energy-efficient maritime transportation: A taxonomy and survey. Transportation Research Part C: Emerging Technologies 26, 331–351.

Qu, R., Burke, E.K., McCollum, B., Merlot, L.T.G., Lee, S.Y., 2009. A survey of search methodologies and automated system development for examination timetabling. Journal of Scheduling 12, 55–89.

Rancourt, M.-È., Cordeau, J.-F., Laporte, G., Watkins, B., 2015. Tactical network planning for food aid distribution in Kenya. Computers & Operations Research 56, 68–83.

Ricca, F., Scozzari, A., Simeone, B., 2011. Political districting: From classical models to recent approaches. 4OR: A Quaterly Journal of Operations Research 9, 223–254.

Robinson, W.S., 1951. A method for chronologically ordering archaeological deposits. American Antiquity 16, 293–301.

Rodgers, J.L., Thompson, T.D., 1992. Seriation and multidimensional scaling: A data analysis approach to scaling asymmetric proximity matrices. Applied Psychological Measurement 16, 105–117.

Rodríguez-Pereira, J., Balçık, B., Laporte, G., Rancourt, M.-È., 2021. Cost sharing mechanisms for multi-country partnerships in disaster preparedness. Production and Operations Management 30, 4541–4565.

Rosa, R.R., Colligan, M.J., 1997. Plain Language about Shiftwork. US Department of Health and Human Services, Cincinnati.

Rossit, D.G., Nesmachnow, S., 2022. Waste bins location problem: A review of recent advances in the storage stage of the municipal solid waste reverse logistic chain. Journal of Cleaner Production 342, 130793.

Sadykov, R., Uchoa, E., Vanderbeck, F., 2020. A generic exact solver for vehicle routing and related problems. Mathematical Programming, 183, 483–523.

Sanguesa, J.A., Torres-Sanz, V., Garrido, P., Martinez, F.J., Marquez-Barja, J.M., 2021. A review on electric vehicles: Technologies and challenges. Smart Cities 4, 372–404.

Santini, A., Schneider, M., Vidal, T., Vigo, D., 2023. Decomposition strategies for vehicle routing heuristics. INFORMS Journal on Computing, forthcoming.

Schiffer, M., 2017. Logistics Networks with Intermediate Stops – Designing Innovative and Green Solutions. Ph.D. thesis. RWTH Aachen University.

Schiffer, M., Klein, P.S., Laporte, G., Walther, G., 2021. Integrated planning for electric commercial vehicle fleets: A case study for retail mid-haul logistics networks. European Journal of Operational Research 291, 944–960.

Schwarzenau, P., Knauth, P., Kiesswetter, E., Brockmann, W., Rutenfranz, J., 1986. Algorithms for the computerised construction of shift systems which meet ergonomic criteria. Applied Ergonomics 17, 169–176.

Seminaroti, M., 2016. Combinatorial Algorithms for the Seriation Problem. Ph.D. thesis. Tilburg University.

Ştefan, A., 1971. Applications of mathematical methods to epigraphy, in: Hodson, F.R., Kendall, D.G., Tăutu, P. (Eds.), Mathematics in the Archaeological and Historical Sciences. Edinburgh University Press, Edinburgh, pp. 267–275.

Toth, P., Vigo, D., 2002. An overview of vehicle routing problems, in: Toth, P., Vigo, D. (Eds.), The Vehicle Routing Problem. SIAM Monographs on Discrete Mathematics and Applications, Philadelphia, pp. 1–26.

Toth, P., Vigo, D. (Eds.), 2014. Vehicle Routing. Problems, Methods, and Applications. DOS-SIAM Series on Optimization, Philadelphia.

Vidal, T., Crainic, T.G., Gendreau, M., Prins, C., 2014. A unified solution framework for multi-attribute vehicle routing problems. European Journal of Operational Research 234, 658–673.

Vidal, T., Laporte, G., Matl, P., 2020. A concise guide to existing and emerging vehicle routing problem variants. European Journal of Operational Research 286, 401–416.

Vuchic, V.R., 2005. Urban Transit: Operations, Planning and Economics. John Wiley & Sons, Hoboken.

Wang, S., Qi, J., Laporte, G., 2022. Governmental subsidy plan modeling and optimization for liquefied natural gas as fuel for maritime transportation. Transportation Research Part B: Methodological 155, 304–321.

Wilkinson, E.N., 1971. Archaeological seriation and the travelling salesman problem, in: Hodson, F.R., Kendall, D.G., Tăutu, P. (Eds.), Mathematics in the Archaeological and Historical Sciences. Edinburgh University Press, Edinburgh, pp. 276–284.

Wøhlk, S., Laporte, G., 2018. A fast heuristic for large-scale capacitated arc routing problems. Journal of the Operational Research Society 69, 1877–1887.

Wøhlk, S., Laporte, G., 2019. A districting-based heuristic for the coordinated capacitated arc routing problem. Computers & Operations Research 111, 271–284.

Wøhlk, S., Laporte, G., 2022. Transport of skips between recycling centers and treatment facilities. Computers & Operations Research 145, 105879.

Yu, H., Sun, X., Solvang, W.D., Laporte, G., Lee, C.K.M., 2020. A stochastic network design problem for hazardous waste management. Journal of Cleaner Production 277, 123566.

Zbib, H., Laporte, G., 2020. The commodity-split multi-compartment capacitated arc routing problem. Computers & Operations Research 122, 104994.

Zhuge, D., Wang, S., Zhen, L., Laporte, G., 2020. Schedule design for liner services under vessel speed reduction incentive programs. Naval Research Logistics 67, 45–62.

Zhuge, D., Wang, S., Zhen, L., Laporte, G., 2021. Subsidy design under a vessel speed reduction incentive program. Naval Research Logistics 68, 344–358.