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Optimal electric vehicle charging with dynamic pricing, customer preferences and power peak reduction

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Abstract : We consider a provider of electric vehicle charging that operates a network of charging stations and uses time-varying pricing to maximize profit and reduce the impact on the electric grid. We propose a bilevel model with a single leader and multiple disjoint followers. The customers (followers) makes decisions independently from each the other. The provider (leader) sets the prices for each station at each time slot, and ensures there is enough energy to charge. The charging choice of each customer is represented by a combination of a preference list of (station, time) pairs and a reserve price. The proposed model takes thus into accounts for the heterogeneity of customers with respect to price sensitivity and charging preferences. We define a single level reformulation based on the reformulation for the rank pricing problem. Computational results highlight the efficiency of the new reformulation and the impact of the model on the grid peak.

Keywords : Electric vehicle charging, dynamic pricing, bilevel optimization, preference list, reserve price

1 Introduction

Due to many political and environmental incentives, the number of electric vehicles (EVs) has increased dramatically in recent years, especially in urban areas. This rapid growth creates a large number of operational challenges for charging service providers. In particular, it is important to manage the location of charging stations (see e.g. [2, 10, 11, 14]), the size of the stations to avoid queues (see e.g. [17]), as well as the impact of charging on the electricity grid (see e.g. [13, 15]). The work of [18] highlights three issues arising from the rapid increase in the number of EVs: a worsening in the peak-to-valley difference grid load leading to a cost increase and a deterioration in the service quality of the networks and installations, a difficulty in meeting the demand of EV customers, and an inequitable distribution of customers among the stations leading to congestion.

The literature devoted to the study of EV charging station management may be classified according to three characteristics: design issues, pricing issues, and joint design & pricing issues. We refer to [12] for a survey. In this work, we focus on the pricing issue.

Pricing is a key element in the distribution of customers: too attractive prices result in: i) queues at charging stations; ii) failure to meet the demand; and iii) high peaks on the distribution grid; on the other hand, too high prices deter customers from charging at this time and location and thus reduce the revenue and create potential future grid imbalances. The authors in [12] define different criteria to characterize dynamic pricing and describe multiple implementations.

In this paper, we address the problem of an EV charging provider that operates a network of charging stations and wishes to apply a dynamic pricing strategy to spread the customers in time and space in order to maximize the profit and reduce the negative grid impacts as well as queues at charging stations. We refer to this problem as $(DPEV)$.

Bilevel models are well suited to represent hierarchical decision-making processes involving two types of decision agents, such as the $DPEV$. Indeed, the charging station managers need to take into account customers' charging decisions when setting the prices. Customers' choice rules can be modeled in a variety of ways. Profit maximization, which takes into account customer preferences, traditionally focuses on maximizing the utility of a consumer. This rule assumes that the customer typically makes a decision by measuring some attributes of the product. However, customers may be partially rational or may fail to evaluate all attributes related to an alternative. In this paper, we consider a non-parametric ranking-based consumer choice model, assuming that each of them possesses its own ranking of the candidate products, yielding an incomplete list of preferences for each customer.

Most papers in the literature related to EV charging pricing problems are based on the utility maximization paradigm. More precisely, they determine the choice of the path of the customers in the network according to travel time, prices, and potential queuing time at stations (see [3, 8, 9, 16] for example). One main difficulty of these works is the evaluation of the attributes related to the path choices and the definition of efficient solution methods to solve large instances due to the large number of potential paths.

An alternative approach was proposed in [1] where the behaviour of EV customers is represented using a preference list and a reserve price (maximum price threshold). This approach makes use of a bilevel optimization formulation in which the upper-level models the charging service provider that seeks to locate, size and price charging stations to maximize its profit, while the lower level models EV customers who individually select the first available charging station from their preference list that meets the customer's reserve price.

In this paper, we define a bilevel optimization model for the $DPEV$ representing the heterogeneous customer decision process as the choice of a location and time to charge in a predefined preference list related to a reserve price [6]. We consider a single EV charging station manager and do not take into account the reaction of the competition. Unlike in [6], the locations and sizes of the charging stations are fixed in advance, and the focus in this paper is on determining the optimal prices and quantities of energy (recharges) to maximize profit, where the profit is the difference between the revenue from providing recharges to customers and the cost of the required energy. We provide an efficient solution approach based on the structure of the problem and the work of [5] for the rank pricing problem. This single level reformulation leads to the solution of a mixed-integer linear optimization problem. We report computational results highlighting the quality of the proposed reformulation with respect to the classical one-level reformulation based on KKT conditions, or to available bilevel optimization software. We also discuss sensitivity analysis results related to the tradeoff between revenue and electrical grid peak reductions.

The remainder of the paper is organized as follows. Section 2 is devoted to a bilevel formulation for the $DPEV$. Section 3 is devoted to single level formulations of the bilevel model. Section 4 includes two illustrative examples. Section 5 is devoted to testing the performance of the models by means of a computational study and Section 6 constitutes a conclusion of the paper.

2 Bilevel model for $DPEV$

Problem $DPEV$ involves two decision-makers interacting sequentially and hierarchically. An electricity provider (the leader) needs to define time-varying prices associated with EV charging stations as well as the quantity of energy allocated to each station, taking into account the preferences of the customers (the followers) with respect to the place and time period to charge.

More precisely, the provider is in charge of a set S of charging stations; each one has a capacity δ_s representing the number of charging spots at the station. The electricity cost for the service provider is denoted by $c_t, \forall t \in T$, where $t \in T$ is a time period. The service provider needs to select the price to charge customers for each pair (station, time). The possible prices belong to a discrete set Π common to all pairs (station, time). We define a charging unit as the complete charge of a vehicle, and the charging time is not taken into account.

The set of customers is denoted by U. Each customer needs exactly to charge one unit during his trip. We introduce a fictitious station s^A that represents the competition to avoid upward pricing due to this constraint. For feasibility reasons, we do not consider capacity constraints for the competition. Each customer has a budget β_u, and we consider that a customer charges in a competing station if the leader's prices exceed his budget. This budget can be seen as a reserve price for the customer.

For fixed price schedules determined by the leader, the customers will select the place and time to charge according to a predefined preference list.

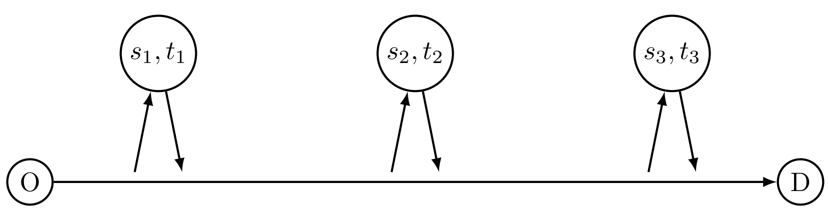

Figure 1: Representation of a customer's route using (station, time) pairs.

The preference list corresponds to an ordered set of station/time pairs where a customer agrees to charge if the leader's price does not exceed his budget. Let R_u be the ordered list of pairs (station, time) for customer u. Customer u prefers to load at pair R_u^ℓ rather than $R_u^{\ell'}$ if $\ell < \ell'$ (ordered in decreasing order of preferences).

For a given pair (s, t), the set $L_{s,t}$ is defined by all pairs (u, ℓ) such that

$$(u, \ell) \in L_{s,t} \text{ iif } R_u^\ell = (s, t) \tag{1a}$$

The set $L_{s,t}$ corresponds to all pairs (u,ℓ) such that the ℓ^{th} choice in the preference list for u is (s,t). For each customer $u \in U$, α_u defines the monetary inconvenience of each pair in his preference list. More precisely, a customer of u undergoes a penalty $\ell\alpha_u$ if he charges at the ℓ^{th} choice of his preference list.

The parameters of the model are summarized in Table 1.

Table 1: Model parameters.

S	set of charging stations
T	time periods
U	set of customers
δ_s	number of charging spots for station $s \in S$
s^A	fictitious station
Π	discrete set of prices
c_t	cost of one unit of energy available at time period $t \in T$ for the leader
R_u	preference list of customer $u \in U$
β_u	budget of customer $u \in U$
α_u	monetary inconvenience for customer $u \in U$
$L_{s,t}$	set of pairs (customer, choice) corresponding to (s,t)

We next introduce three sets of leader's and followers' decision variables. Let x_s^t be the number of charging units available (at cost c_t) for station s at time t. The variables $y_u^{\ell,p}, \forall u \in U, \ell \in \{0,...,|R_u|-1\}, p \in \{1,...,|\Pi|\}$ are binary and $y_u^{\ell,p} = 1$ if customer u charges at station and time corresponding to his ℓ^{th} choice at the p^{th} price. Finally, variables $\theta_p^{s,t}, \forall p \in \{1,...,|\Pi|\}, s \in S, t \in T$ are binary and $\theta_p^{s,t} = 1$ if the price of station s at time t is fixed to Π_p. $DPEV$ can be formulated as:

$$\textbf{DPEV} : \max_{x,y,\theta} \sum_{u \in u} \sum_{\ell=0}^{|R_u|-1} \sum_{p=1}^{|\Pi|} \Pi_p y_u^{\ell,p} - \sum_{t \in T} \sum_{s \in S} x_s^t c_t \tag{2a}$$

$$\text{s.t.} \sum_{p=1}^{|\Pi|} \theta_p^{s,t} = 1 \qquad \forall (s,t) \in S \times T \tag{2b}$$

$$\sum_{(u,\ell) \in L_{s,t}} \sum_{p=1}^{|\Pi|} y_u^{\ell,p} \leq x_s^t \qquad \forall (s,t) \in S \times T \tag{2c}$$

$$\sum_{(u,\ell) \in L_{s,t}} \sum_{p=1}^{|\Pi|} y_u^{\ell,p} \leq \delta_s^t \qquad \forall (s,t) \in S \times T \tag{2d}$$

$$x_s^t \in \mathbb{N}, \qquad \forall (s,t) \in S \times T \tag{2e}$$

$$\theta_p^{s,t} \in \{0,1\}, \qquad \forall (s,t) \in S \times T, p \in \{1,...,|\Pi|\} \tag{2f}$$

For each $\tilde{u} \in U$:

$$\min_{y_{\tilde{u}}} \sum_{p=1}^{|\Pi|} \sum_{\ell=0}^{|R_{\tilde{u}}|-1} y_{\tilde{u}}^{\ell,p} (\Pi_p + \ell * \alpha_{\tilde{u}}) + y_{\tilde{u}}^{s^A} \beta_{\tilde{u}} \tag{2g}$$

$$\sum_{\ell=0}^{|R_{\tilde{u}}|-1} \sum_{p=1}^{|\Pi|} y_{\tilde{u}}^{\ell,p} + y_{\tilde{u}}^{s^A} = 1 \tag{2h}$$

$$y_{\tilde{u}}^{\ell,p} \leq \theta_p^{s(\ell),t(\ell)}, \qquad \forall \ell \in \{0,...,|R_{\tilde{u}}|-1\}, p \in \{1,...,|\Pi|\} \tag{2i}$$

$$y_{\tilde{u}}^{\ell,p} (\Pi_p + \ell * \alpha_{\tilde{u}}) \leq \beta_{\tilde{u}}, \qquad \forall \ell \in \{0,...,|R_{\tilde{u}}|-1\}, p \in \{1,...,|\Pi|\} \tag{2j}$$

$$y_{\tilde{u}}^{\ell,p} \in \{0,1\}, \qquad \forall \ell \in \{0,...,|R_{\tilde{u}}|-1\}, p \in \{1,...,|\Pi|\} \tag{2k}$$

At the upper level, the leader maximizes his profit by determining the prices $\theta_p^{s,t}$ and the quantities of energy x_s^t for each time $t \in T$ and each station $s \in S$. The Profit is the difference between revenue from energy sales to customers and the cost of energy purchases.

Constraints (2b) define a single price for each pair (station, time). The last constraints are capacity constraints, with respect to the demand (2c) and the number of available spots (2d) in the charging stations. At the lower level, each customer selects the pair in his preference list, minimizing the total cost of charging, including the penalty for the inconvenience due to the order of the choices in the list.

Constraint (2h) defines a customer's choice for charging at a station from the leader or to the competition. Constraints (2i) ensure that customers' choices are consistent with the prices set by the leader. Note that the budget constraints (2j) are redundant with respect to the objective function. Nevertheless, they will be useful for the next reformulations.

DPEV is a linear bilevel optimization problem. One of the particularities **DPEV** is the definition of constraints (2c) and (2d) at the upper level. The leader sets prices, ensuring the followers' capacity is satisfied. Note that in the case of the non unicity of the solution of the followers for fixed leader's decisions, we assume that the followers select the solution leading to the highest objective function value of the leader. Thus, we consider the optimistic version of a bilevel optimization problem.

To take into account the impact of the grid peak consumption on the pricing decisions, we introduce a new term in the leader's objective function. We first define X_0 as the maximum peak consumption value obtained in the optimal solution of the model **DPEV**.

The new leader's objective function is then given by:

$$\max_{x,y,\theta,X} \sum_{u\in u} \sum_{\ell=0}^{|R_u|-1} \sum_{p=1}^{|\Pi|} \Pi_p y_u^{\ell,p} - \sum_{t\in T}\sum_{s\in S} x_s^t c_t + k(X_0 - X)$$

where X is defined as the maximum peak consumption and defined by the next constraint:

$$X \geq \sum_{s\in S} \sum_{(u,\ell)\in L_{s,t}} \sum_{p=1}^{|\Pi|} y_u^{\ell,p} \quad \forall t \in T$$

The leader is rewarded for lowering the maximum peak X, but has only indirect control over this peak because it arises from the customers' choices. We call this model $\mathbf{DPEV}_k$. The parameter k expresses all terms in the same unit. It can also be used as a weighting parameter in the objective function.

$$\mathbf{DPEV}_k : \max_{x,y,\theta,X} \sum_{u\in u} \sum_{\ell=0}^{|R_u|-1} \sum_{p=1}^{|\Pi|} \Pi_p y_u^{\ell,p} - \sum_{t\in T}\sum_{s\in S} x_s^t c_t + k(X_0 - X) \tag{3a}$$

$$\text{s.t.} \quad X \geq \sum_{s\in S} \sum_{(u,\ell)\in L_{s,t}} \sum_{p=1}^{|\Pi|} y_u^{\ell,p} \quad \forall t \in T \tag{3b}$$

$$(2b), ..., (2k)$$

$$X \in \mathbb{R} \tag{3c}$$

3 Single-level reformulations of DPEV

In this section, we first present a new single-level MIP formulation of **DPEV** based on the work of [5] for the rank pricing problem (RPP). Next, we remind classical single-level reformulations of bilevel models to be used in the numerical results section.

3.1 Linear single level reformulation

We define for each customer $u \in U$ an order relation $\preceq_u$ which for two pairs $(\ell, p), (\ell', p'), \ell, \ell' \in \{0, ..., |R_u| - 1\}$ and $p, p' \in \{1, ..., |\Pi|\}$ defines the most beneficial one for the customer. The order relation is given by

$$(\ell, p) \preceq_u (\ell', p') \text{ if } \Pi_p + \ell\alpha_u \geq \Pi_{p'} + \ell'\alpha_u.$$

The set of pairs (ℓ', p') preferred to (ℓ, p) for a customer u is defined by:

$$\mathcal{B}_{u,\ell,p} = \{(\ell', p'), \ell' \in R_u | (\ell, p) \preceq_u (\ell', p')\}. \tag{4}$$

Finally, we define an indicator of whether the competition is more beneficial than the pair (ℓ,p) or not.

$$\forall u \in U, \ell \in R_u, p \in \{1, ..., |\Pi|\}, C_{u,l,p} = \begin{cases} 1, & \text{if}(\ell * \alpha_u) + \Pi_p \geq \beta_u \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

Proposition 1. *For each customer u, a feasible vector y_u is optimal for the lower-level problem if the following constraints are satisfied:*

$$\sum_{(\ell',p') \in \mathcal{B}_{u,\ell,p}} y_u^{\ell',p'} + C_{u,\ell,p} y_u^{s^A} \geq \theta_p^{s(\ell),t(\ell)}, \qquad \forall \ell \in \{0, ..., |R_{\tilde{u}}| - 1\}, p \in \{1, ..., |\Pi|\}. \tag{6}$$

Proof. Let y_u be a feasible vector satisfying the constraints. Since y_u is a feasible solution for customer u, we know that $\exists!(\ell, p)$ such as $y_u^{\ell,p} = 1$ (constraint 2h). To obtain a contradiction, suppose that y_u is not optimal for customer u, then $\exists(\ell^*, p^*)$ such as $(\Pi_{p^*} + \ell^* \times \alpha_u) < (\Pi_p + \ell \times \alpha_u)$ (given by the objective function) and $\theta_{p^*}^{s(\ell^*),t(\ell^*)} = 1$ (given by 2i). By 4, $(\ell, p) \notin \mathcal{B}_{u,\ell^*,p^*}$. As a consequence the constraint corresponding to (ℓ^*, p^*) is not respected:

$$\sum_{(\ell',p') \in \mathcal{B}_{u,\ell^*,p^*}} y_u^{\ell',p'} + C_{u,\ell^*,p^*} y_u^{s^A} = 0 \text{ and } \theta_{p^*}^{s(\ell^*),t(\ell^*)} = 1$$

□

Using Proposition 1, the single-level reformulation is:

$$\mathbf{DPEV^{SL}} : \max_{x,y,\theta} \sum_{u \in u} \sum_{\ell=0}^{|R_u|-1} \sum_{p=1}^{|\Pi|} \Pi_p y_u^{\ell,p} - \sum_{t \in T} \sum_{s \in S} x_s^t c_t \tag{7a}$$

$$\text{s.t.} \qquad (2b), (2c), (2d), (2h), (2j)$$

$$\sum_{(u,\ell) \in L_{s,t}} y_u^{\ell,p} \leq \theta_p^{s,t} |U| \qquad \forall (s,t) \in S \times T, p \in \{1, ..., |\Pi|\} \tag{7b}$$

$$\sum_{(\ell',p') \in \mathcal{B}_{u,\ell,p}} y_u^{\ell',p'} + C_{u,l,p} y_u^{s^A} \geq \theta_p^{s(\ell),t(\ell)}, \qquad \forall u \in U, \ell \in R_u, p \in \{1, ..., |\Pi|\} \tag{7c}$$

$$x_s^t \in \mathbb{N} \tag{7d}$$

$$\theta_p^{s,t} \in \{0, 1\}, \qquad \forall (s,t) \in S \times T, p \in \{1, ..., |\Pi|\} \tag{7e}$$

$$y_u^{\ell,p} \in \{0, 1\} \qquad \forall u \in U, \ell \in \{0, ..., |R_u| - 1\}, p \in \{1, ..., |\Pi|\} \tag{7f}$$

Optimality conditions for the lower level are given by (7c). Constraint (7b) ensures consistency between the prices set by the leader and the prices chosen by the customers. Constraint (7b) corresponds to the aggregation of constraints 2i.

Problem $\mathbf{DPEV^{SL}}$ can be solved with a mixed integer linear solvers. As we have not added a constraint in the follower problem, we can use the same reformulation for the problem ($\mathbf{DPEV}_k$).

$$\mathbf{DPEV_k^{SL}} : \max_{x,y,\theta,X} \sum_{u \in u} \sum_{\ell=0}^{|R_u|-1} \sum_{p=1}^{|\Pi|} \Pi_p y_u^{\ell,p} - \sum_{t \in T} \sum_{s \in S} x_s^t c_t + k(X_0 - X) \tag{8a}$$

$$\text{s.t.} \qquad X \geq \sum_{s \in S} \sum_{(u,\ell) \in L_{s,t}} \sum_{p=1}^{|\Pi|} y_u^{\ell,p} \quad \forall t \in T \tag{8b}$$

$$(2b), (2c), (2d), (2h), (2j), (7b), (7c), (7d), (7e), (7f)$$

$$X \in \mathbb{R} \tag{8c}$$

3.2 Classical single level reformulation based on KKT optimality conditions

In this section, we present a classical reformulation of a bilevel optimization problem as a single level one based on the Karush-Kuhn-Tucker optimality conditions. We first prove that the linear relaxation of the lower-level problem is equivalent to the initial one.

Proposition 2. *For each follower problem SP_u, the linear relaxation is equivalent to the initial problem.*

Proof. Each sub-problem can be formulated as a min-cost flow problem in a graph. The first level variables $\theta^{\ell,p}$ represent the arc capacity. The cost of an arc is given by its contribution to the objective function. A final arc, of capacity 1, represents the competition. Since the constraint matrix of this type of problem is totally unimodular, the optimal solution of the linear relaxation is always integer. □

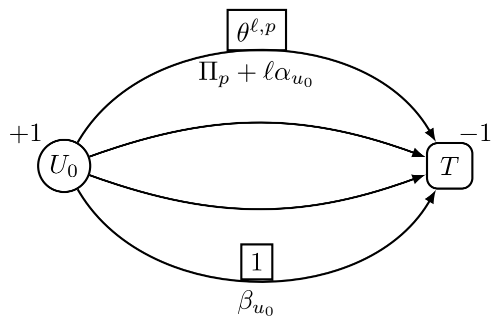

Figure 2: Representation of the follower problem as a min-cost flow problem.

The KKT optimal conditions can replace each follower sub-problem. In this approach, the lower level's optimality conditions (stationarity, primal feasibility, dual feasibility, complementary slackness) are added to the upper level to yield an equivalent single-level formulation. Linearization techniques need to be applied to obtain a mixed integer linear program.

The second-level variable integrality constraints are next reintroduced into the leader's problem. Since the lower-level problem is totally unimodular, there are always optimal lower-level solutions that meet the integrality requirements, and these, according to the 'optimistic' assumption, can be selected by the leader. In the following, we describe the conditions obtained for one customer. Recall that the sub-problem for customer $\tilde{u}$ is given by:

$$\min_{y_{\tilde{u}}} \sum_{p=1}^{|\Pi|} \sum_{\ell=0}^{|R_{\tilde{u}}|-1} y_{\tilde{u}}^{\ell,p}(\Pi_p + \ell * \alpha_{\tilde{u}}) + y_{\tilde{u}}^{s^A} \beta_{\tilde{u}} \tag{9a}$$

$$\sum_{\ell=0}^{|R_{\tilde{u}}|-1} \sum_{p=1}^{|\Pi|} y_{\tilde{u}}^{\ell,p} + y_{\tilde{u}}^{s^A} = 1 \tag{9b}$$

$$y_{\tilde{u}}^{\ell,p} \leq \theta_p^{s(\ell),t(\ell)}, \quad \forall \ell \in \{0, ..., |R_{\tilde{u}}| - 1\}, p \in \{1, ..., |\Pi|\} \tag{9c}$$

$$y_{\tilde{u}}^{\ell,p}(\Pi_p + \ell * \alpha_{\tilde{u}}) \leq \beta_{\tilde{u}}, \quad \forall \ell \in \{0, ..., |R_{\tilde{u}}| - 1\}, p \in \{1, ..., |\Pi|\} \tag{9d}$$

$$y_{\tilde{u}}^{\ell,p} \geq 0, \quad \forall \ell \in \{0, ..., |R_{\tilde{u}}| - 1\}, p \in \{1, ..., |\Pi|\} \tag{9e}$$

$$y_{\tilde{u}}^{s^A} \geq 0 \tag{9f}$$

Remark 1. *Integrality constraints are replaced by (9e). The condition $y_{\tilde{u}}^{\ell,p} \leq 1$ is induced by (9b) and (9e).*

The dual feasibility, complementary slackness, and stationarity are given by:

Complementary slackness:

$$(y_{\tilde{u}}^{\ell,p} - \theta_p^{s(\ell),t(\ell)})\mu_{\tilde{u}}^{\ell,p} = 0 \qquad \forall \ell \in \{0,...,|R_{\tilde{u}}|-1\}, p \in \{1,...,|\Pi|\} \tag{10a}$$

$$(y_{\tilde{u}}^{\ell,p}(\Pi_p + \ell * \alpha_{\tilde{u}}) - \beta_{\tilde{u}})\phi_{\tilde{u}}^{\ell,p} = 0 \qquad \forall \ell \in \{0,...,|R_{\tilde{u}}|-1\}, p \in \{1,...,|\Pi|\} \tag{10b}$$

$$y_{\tilde{u}}^{\ell,p}\Delta_{\tilde{u}}^{\ell,p} = 0 \qquad \forall \ell \in \{0,...,|R_{\tilde{u}}|-1\}, p \in \{1,...,|\Pi|\} \tag{10c}$$

$$y_{\tilde{u}}^{s^A}\Delta_{\tilde{u}}^{s^A} = 0 \tag{10d}$$

Stationarity:

$$\Pi_p + \ell * \alpha_{\tilde{u}} + \lambda_{\tilde{u}} + \mu_{\tilde{u}}^{\ell,p} + \phi_{\tilde{u}}^{\ell,p}(\Pi_p + \ell * \alpha_{\tilde{u}}) - \Delta_{\tilde{u}}^{\ell,p} = 0 \qquad \forall \ell \in \{0,...,|R_{\tilde{u}}|-1\}, p \in \{1,...,|\Pi|\} \tag{11a}$$

$$\lambda_{\tilde{u}} - \Delta_{\tilde{u}}^{s^A} + \beta_{\tilde{u}} = 0 \tag{11b}$$

$$\lambda_{\tilde{u}} \in \mathbb{R} \tag{11c}$$

$$\mu_{\tilde{u}}^{\ell,p}, \phi_{\tilde{u}}^{\ell,p}, \delta_{\tilde{u}}^{\ell,p} \geq 0 \qquad \forall \ell, p \tag{11d}$$

$$\Delta_{\tilde{u}}^{s^A} \geq 0 \tag{11e}$$

The resulting problem is not linear because of the complementarity slackness. These constraints can be linearized using the big-M approach or solved directly using SOS1 constraints [4].

4 Illustrative examples

To give insight into the solution structure, we illustrate the impact of the model on small instances.

4.1 Impact of reserve price and competition

Let us consider a first example involving three customers where only one customer will be able to charge during each time slot.

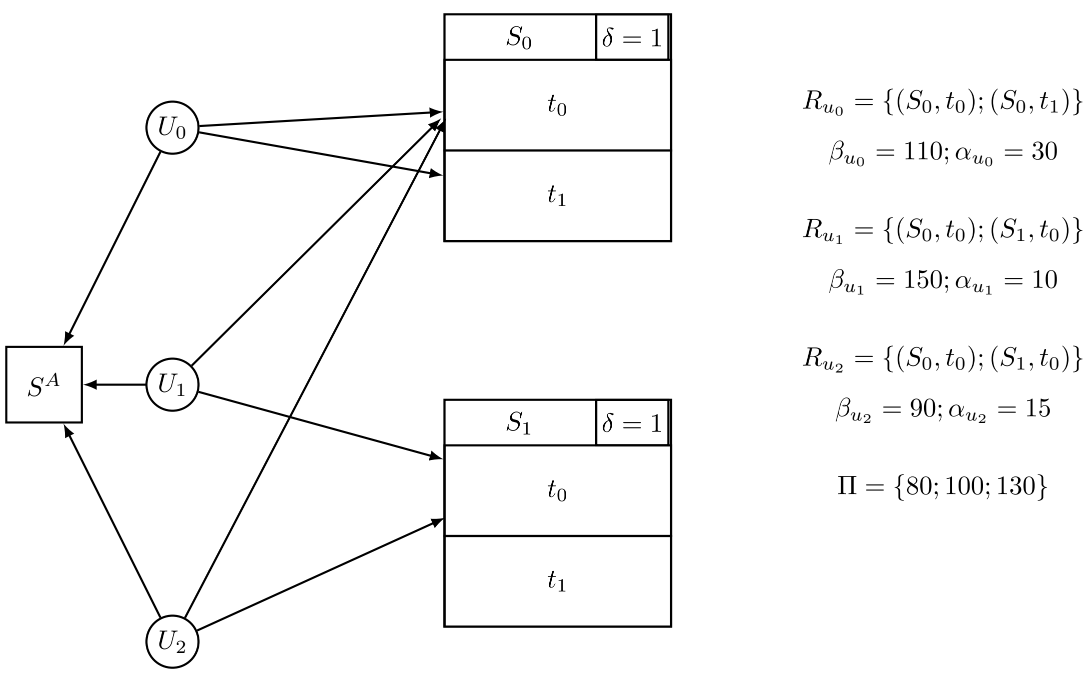

Figure 3: Illustration of an example with 3 customers with a preference list of 2 choices.

To capture customer u_2 (with a low budget), the price needs to be smaller than 80, which corresponds to the pair (S_0, t_0). This pair is also the preferred choice of the other two customers. As the number of chargers $\delta = 1$, it is impossible to set a price equal to 80 otherwise all customers will charge at (S_0, t_0). Customer u_2 needs thus to go to the competing station in order to allow larger prices and higher revenue for the other customers. Customers u_0 and u_1 have the same first choice in their preference list but a different second choice. The prices need to be defined in such a way that at least one of them charges at his second choice. The optimal solution is given by :

- $\theta_2^{0,0} = \theta_0^{0,1} = \theta_2^{1,0} = \theta_0^{1,1} = 1$
- $y_0^{1,0} = y_1^{1,2} = y_2^{s^A} = 1$

and depicted in Figure 4. Note that this solution corresponds to an optimistic solution to the problem: the customer U_0 can charge at a competing station for the same price, but this solution is less advantageous for the leader.

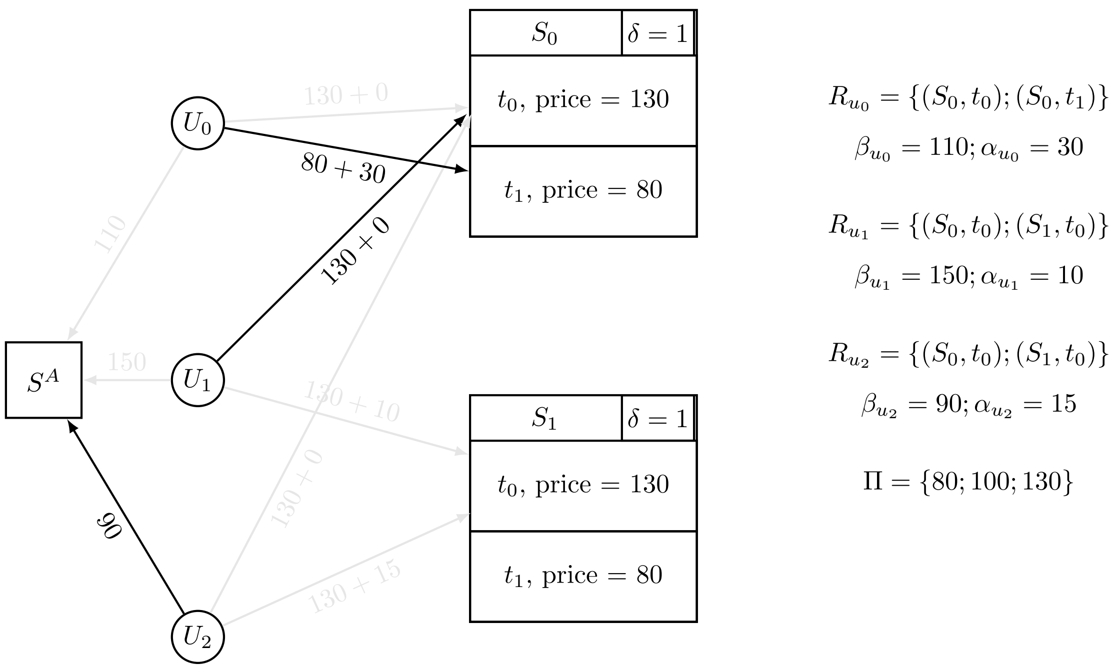

Figure 4: Optimal solution of each customer with optimal prices.

4.2 Impact of capacity constraints at the first level

In some cases, bilevel feasibility is not guaranteed when pricing strategies do not lead to a distribution of customers in accordance with the charging stations' capacity constraints. More precisely, as the capacity limits are leader's constraints, customers are not concerned with available spots at charging stations when they select their time and location charging pair. We illustrate how to remedy this drawback in the following example involving identical preference lists for all customers.

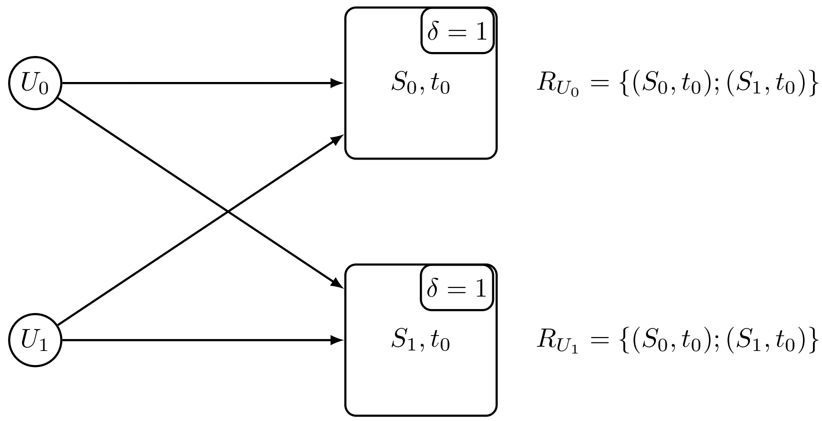

Figure 5: Bilevel infeasible example with two customers.

One way to guarantee bilevel feasibility is to add a very high price that makes a station/time pair unattractive to all customers. All the customers will then charge at the competition leading to a feasible solution.

Proposition 3. *If* $\exists p \in \{0, ..., |\Pi|\}, \Pi_p > \max_{u \in U} \beta_u$ *then problem* **DPEV** *is bilevel feasible.*

Proof. If all prices are set at $\max_{u \in U} \beta_u$, all customers will charge at the competition, and capacity constraints will be satisfied. This solution is always bilevel feasible. □

This price can be interpreted as the closure of a charging station.

5 Computational experiments

In this section, we compare the performance of 4 solution approaches for $DPEV$ on randomly generated instances and provide an extensive sensitivity analysis with respect to the parameter k associated with the maximum consumption peak in the leader's objective function.

5.1 Instances and methods definition

We first describe the instance generation process. The number of stations and times are defined in such a way that the number of available units of energy is greater than the number of customers, i.e., $S \times T \times \delta > |U|$. Even if it is realistic to assume that the provider can satisfy all the demand, this condition is not necessary, since feasibility is guaranteed in the worst case by Proposition 3.

We have generated instances ranging from 10 to over 5,000 customers.

The customer parameters are chosen randomly such that each customer possesses at least one charging option, satisfying the budget constraint (each customer's budget is greater than the lowest price from the set of possible prices).

We define probabilities to the (station, time) pairs selection process to avoid homogeneous distribution. In other words, some (station, time) pairs are more attractive for several customers and have a greater chance of being selected.

We next compare the performance of 4 solution approaches for $DPEV$ (without the penalty on the grid peaks). The first one consists in applying the generic bilevel solver (GBS) proposed by [7] on the bilevel optimization formulation. The second and third methods consist in solving the MIP reformulation based on the Karush-Kuhn-Tucker presented in Section 4 using two types of linearization techniques: the linearization based on big M (KKT BIGM), and the linearization using SOS1 constraints (KKT SOS). Finally the last one consists in solving the single level reformulation defined in Section 3. Tests were performed on Intel Core i5-8350U 1.70 GHz 8GB for results on Table 2 and on a Mac Book Pro M1 with 16 Gb and CPLEX 20.1 as a MIP solver for the other.

5.2 Efficiency on solution methods on model DPEV

This section is focused on the study of the solution methods to solve model **DPEV**. For simplicity reasons we do not consider the term related to the peak in the leader's objective function. We first compare in Table 2 the results obtained by solving the single level formulation $\mathbf{DPEV^{SL}}$ defined in Section 3 by the MIP with the results obtained with a generic bilevel solver GBS.

Table 2: Comparison of the results obtained with our reformulation $\mathrm{DPEV^{SL}}$ and a generic bilevel solver GBS. The number of instances solved to the optimum in 1 hour and the average solving time for 5 instances solved to the optimum are given for instances with each of 10, 20, and 30 customers.

Instances	$\mathbf{DPEV^{SL}}$		GBS	
#customers	#OPT (1h)	Mean Time (solved)	#OPT (1h)	Mean Time (solved)
10	5 / 5	0.04	5 / 5	6.22
20	5 / 5	0.03	4 / 5	628.91
30	5 / 5	0.03	2 / 5	1989.58

The bi-level solver is only able to solve instances with up to 30 customers in less than 1 hour. These experiments do not question the quality of the GBS but show the complexity to solve **DPEV**. Solving the reformulation $\mathbf{DPEV^{SL}}$ leads to better performance and underlines the importance of taking the structure of the problem into account when designing solution methods. Note that both methods determine optimal solutions.

Table 3 shows the results obtained with the reformulation based on KKT conditions.

Table 3: Comparison of the results obtained with our reformulation $\mathrm{DPEV^{SL}}$, KKT reformulation using BigM, and KKT reformulation using SOS1 constraints. All instances are solved to the optimum in 1 hour. The average solving time and the standard deviation are given for instances with 10, 20, and 30 customers (10 instances for each category) and 50 customers (30 instances).

	$\mathbf{DPEV^{SL}}$		KKT BigM		KKT SOS1	
	Mean Time (sec)	SD	Mean Time (sec)	SD	Mean Time (sec)	SD
10	0.02	0.01	0.05	0.02	0.18	0.18
20	0.03	0.02	0.09	0.02	0.96	1.92
30	0.04	0.02	0.22	0.11	2.29	1.90
50	0.06	0.02	0.39	0.15	1285.55	927.57

The computation time of solution of **DPEV** with KKT reformulations using SOS1 constraint are drastically more significant than those of KKT reformulation with BigM. However, they ensure the accuracy of the method. Indeed, the results obtained using BigM may be unfeasible if the value of BigM is too large (or non-optimal if it is too small). Bounds on BigM are difficult to compute due to their link with the dual variables. In all cases, the reformulation defined in Section 3 leads to better results than the KKT reformulations. Table 4 reports the results of tests carried out on the largest instances and supports this observation.

Finally, we present in Table 5 the results obtained on very large instances with up to 5000 customers. These results show that instances involving thousands of customers can be solved using the reformulation presented in Section 3. This highlights the strong potential for this approach, going beyond the performances obtained using classic reformulation approaches from the literature.

Table 4: Comparison of the results obtained with our reformulation $\mathbf{DPEV^{SL}}$ and KKT reformulation (using BigM). The number of instances solved to the optimum in 30 minutes and the average solving time for instances solved to the optimum is given for instances with 100, 200, and 500 customers (20 instances for each category).

	$\mathbf{DPEV^{SL}}$		KKT BigM	
	#OPT	Mean Time (sec)	#OPT	Mean Time (sec)
100	20/20	0.16	20/20	4.79
200	20/20	0.44	20/20	11.70
500	20/20	2.28	20/20	242.04

Table 5: Number of instances solved to the optimum with a time limit of one hour, average time for instances solved to the optimum, and average number of clients served in the optimal solution for instances with 2000, 3000, 4000, and 5000 customers using our reformulation $\mathbf{DPEV^{SL}}$.

# customers	# Optimal	Mean Time (sec)	# customers served
2000	15/15	33.16	796.33
3000	15/15	71.65	1242.13
4000	10/10	141.09	2116.3
5000	10/10	216.08	2624.1

5.3 Impact of k

We next study the impact of the factor k on the leader's and follower's objective function values as well as on the structure of the solutions. We compare the solution obtained with $\mathbf{DPEV_k^{SL}}$ to the basic case where the prices are the same for each station, and the customers charge at the (station, time) pair in the first position in their preference list. This case called *static* in the following, generates the maximum consumption peak. Note that in the static case, the number of customers recharging at a station/time pair can be higher than the maximum possible number of customers allowed to charge, ie. charging station capacity is not considered in the greedy solution. The impact of the factor k on the leader's profit is illustrated in Figure 6, and the impact on the maximum consumption peak in Figure 7, on a set of 30 instances of 500 customers.

In Figure 6, we report the optimal net profit (without penalty cost) on a set of instances with 500 customers. We solve model $\mathbf{DPEV^{SL}}$ and model $\mathbf{DPEV_k^{SL}}$ using 3 values of k. We also report values of the maximum peak in Figure 7 for the same configuration.

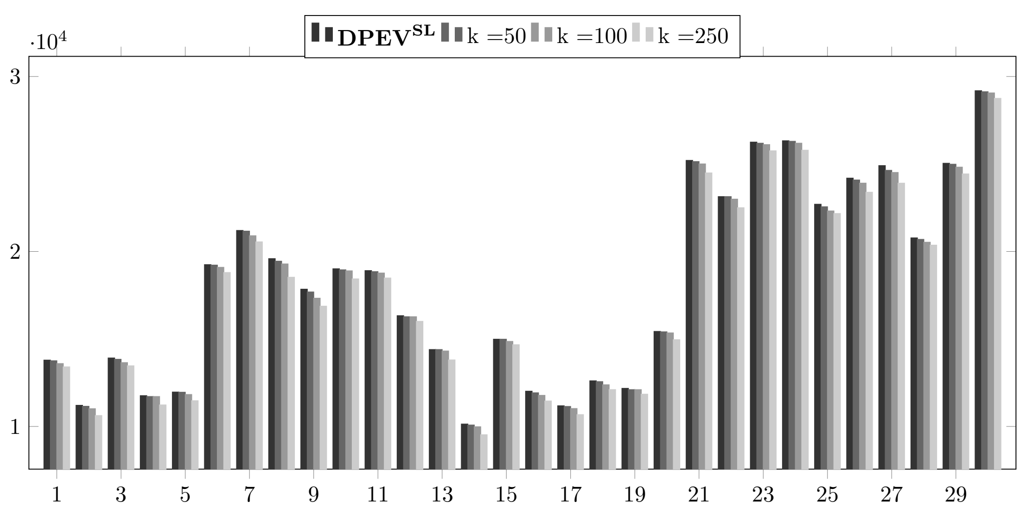

Figure 6: Net profit obtained for 30 instances of 1000 customers, with different values of parameter k.

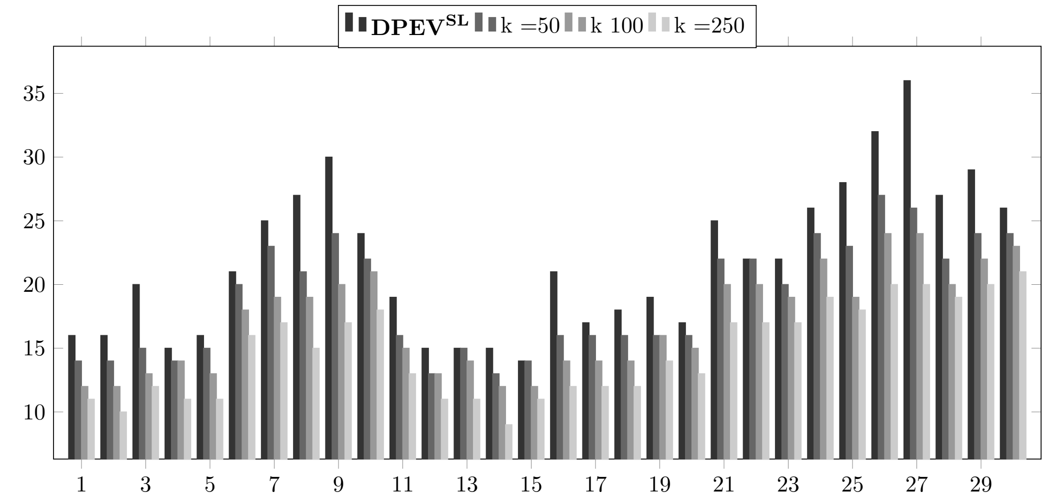

Figure 7: Maximum peak obtained for 30 instances of 1000 customers, with different values of parameter k.

The parameter k has a relatively light impact on the decrease of the leader's profit with an average decrease of 3.32% between the profit obtained with $\mathbf{DPEV^{SL}}$ and $\mathbf{DPEV_k^{SL}}$ using $k = 250$.

On the other hand, we observe a significant reduction of the maximum peak, with an average reduction of 31.30% when the parameter k increases . For some instances, the optimal solutions given by $\mathbf{DPEV^{SL}}$ and by $\mathbf{DPEV_k^{SL}}$ with $k = 50$ are identical. It may either be due to the fact that the solution obtained for $\mathbf{DPEV^{SL}}$ corresponds to a solution that is well distributed between time slots or due to too small the penalty impact on the solution. These results highlight that it is possible to significantly reduce the maximum peak, which is a key element in grid operations, without excessively degrading the benefit. Figure 8 depicts the maximum peaks of the optimal solutions of $\mathbf{DPEV^{SL}}$ and $\mathbf{DPEV_k^{SL}}$ with different k compared with the static case.

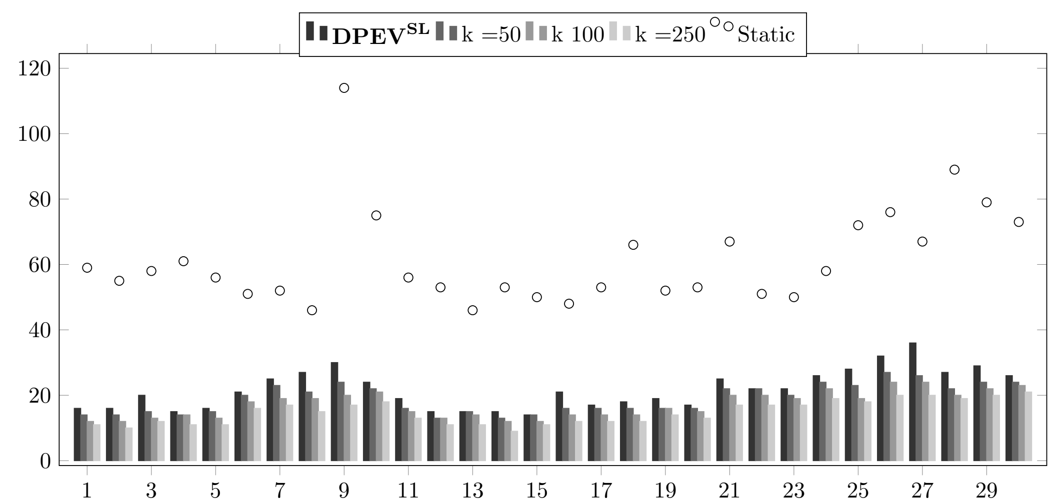

Figure 8: Maximum peak obtained for 30 instances of 1000 customers, with different values of parameter k.

The static case does not represent a bilevel feasible solution but put into highlight the dynamic pricing approach. Indeed, we observe that the first choice of customers generates significant consumption peaks. Dynamic pricing makes it possible to distribute customers across the different (station, time) pairs and thus smooth out the demand over time and space.

6 Conclusion and future resarch

In this paper, we present an original model for an electric vehicle charging station pricing problem that takes customer preferences into account. Several single level reformulations are provided. The linear reformulation based on [5] lead to a solution approach able to solve large instances and shows better results than the classical methods. The results put also into highlight that the maximum consumption peaks can be reduced by slightly degrading the profit of the provider.

In future work, we plan to add energy storage (such as batteries) to the charging stations together with renewable energy generation, either locally at the stations or as an investment by the charging provider in the grid. More generally, EV charging has potential to provide grid services to mitigate the stochasticity of renewable generation and of demand.

Data availability statement The data that support the findings of this study are available from the corresponding author, MFA, upon reasonable request.

Conflict of interest statement The authors confirm that there are no relevant financial or non-financial competing interests to report.

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