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A fast solution approach to solve the generator maintenance scheduling and hydropower production problems simultaneously

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Abstract : The Generator Maintenance Scheduling Problem (GMSP) is a problem that combines a hydropower optimization problem with a scheduling problem. Both problems are known to be hard to solve and combining them leads to an even more challenging mathematical problem. Since the hydropower production functions are nonlinear, hyperplane curve fitting is used to linearize each power production function. The goal of the GMSP is to find an optimal schedule plan to decide when to shut down generators for maintenance. Therefore, one production function needs to be formulated per generator combinations leading to a rather large number of constraints. This paper demonstrates that the complexity of the problems is linked to the number of hyperplanes selected to formulate the power production functions. To accelerate the resolution of the problem, a new heuristic based on the mean square algorithm is presented to reduce the number of hyperplanes required. This heuristic substantially reduces the number of constraints and the solving time is almost ten times faster. Numerical results show that the energy produced and the generated maintenance plannings are similar for both mathematical formulations, more precisely with one hyperplane for each generator combination versus a reduced number of hyperplanes.

Keywords: Hydropower scheduling problem, maintenance scheduling problem, hydropower production functions, stochastic programming, mixed integer linear programming, heuristic

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1 Introduction

This paper is concerned with the Generator Maintenance Scheduling Problem (GMSP). It combines a hydropower optimization problem with a maintenance scheduling problem, both of which are hard to solve in large-scale models because of the large size, integer variable, and nonlinearity of the hydropower production functions. Therefore, containing both problems in a real application setting is a challenge. The current literature covers many hydropower problems or maintenance planning problems, but few papers consider both problems simultaneously. We propose an approach that allows to solve both problems at once. From an industrial point of view, it is important to link the two models because the number of active generators in a plant correlates strongly with plant efficiency. Indeed, the efficiency curve of a power plant is different for each combination of generators and it is possible to produce the same power with fewer generators but the discharged water amount will be different. Also, at times when power demand decreases, it is better to shut down a generator. Actually, if some generators are working at their maximum efficiency the plant can produce the same amount of power with less water than if it worked with more generators. In this case the maintenance can be done by a free indirect costs because the generators need to be shutdown. This approach presents a fast solving time with a good reliability regarding to nonlinear characteristics formulations. Indeed, this problem is formulated as a Mixed-Integer Linear Problem (MILP) but the models still represents the key elements of nonlinear hydropower production functions thanks to the linearization through hyperplanes formulations. Moreover, a heuristic is used to estimate the final power production with a reduced number of hyperplanes. This methods shows similar results as the method without the heuristic, but it is faster.

1.1 Related works

The two problems contained in the GMSP are commonly solved independently. Some authors [1, 2] proved that the two problems are linked and that maintenance planning influences hydropower production. The short-term hydropower optimization is used to determine the water flows of each turbine in each power plant. It usually maximizes the energy production or minimizes production cost and uses precise formulations of the power production functions. The latter are nonlinear and non-convex, it makes the problem hard to solve if a global optimal solution is needed. The variables are discharged water amount, spilled water amount and number of active turbines, the latter case leading to a mixed integer program. Certain authors model this problem as a mixed integer linear program by using some linearizing techniques for the nonlinear power production functions [15, 20, 23, 24]. These production functions depend on reservoir volume or net water head (which is the difference between the forehead and tailrace elevation minus the losses caused by friction in the penstock), and total water discharged or more precisely, water processed and spilled by each turbine and each power plant. In the literature, the short-term hydropower optimization problem is often seen as deterministic. Indeed, the planning horizon is short and the forecasting is commonly seen as deterministic, but some authors have shown that stochastic models lead to increased performance [3]. Many formulations of the problem are possible. For instance, in [4] a stochastic model to fomulate uncertainty of electricity prices in the Norwegian electricity market is presented. A multistage mixed integer linear stochastic model is used to solve the problem. Whereas in [5] an inflow scenario tree approach is presented, also the head variations and nonlinear/nonconvex relationship between discharge and power output are considered. In [6] the authors describe a two-stage scenario tree to solve the short-term hydropower problem. They model electricity load and inflow uncertainty. They propose operational rules to manage the real time dispatchers and their approach increases the value of the objective function by up to 54%. If uncertainty in future prices and inflows are taken into account, it demonstrates and quantifies the benefits of using stochastic models rather than a deterministic model with a nonlinear modelisation [7].

The maintenance scheduling problem is a common study problem which consists in calculating a maintenance calendar for an equipment or a plant by taking into account the profit or efficiency loss of shutting down the system while doing the maintenance. The maintenance problem should take into account workforce costs, materials available and tools costs; these are the direct costs of the maintenance. On the other hand, indirect maintenance costs are usually startup and shutdown costs and production losses [10]. Both costs illustrate that the maintenance has to be scheduled at the right time to have a minimal financial impact. It is an essential element of all industrial sectors. It is possible to increase the efficiency of a system with a good maintenance plan by reducing breakdown risks and emergency maintenance [11]. Emergency maintenances are more costly than preventive maintenances since the outage times of the equipment are longer. Also, breakdowns can occur on days when the maintenance costs are more expensive [12]. A breakdown can also happen a day where the maintenance cost is more expensive. Finally, maintenance for a broken piece in an industrial machinery can bring other defects or other broken pieces that can increase the total cost of the maintenance. For all these reasons, it is mandatory to have a good maintenance plan. Such a plan respects all constraints defined by the producer and schedules preventive maintenance before breakdowns occur.

The literature contains a few studies that link an industrial production problem with a maintenance scheduling problem. In the specific case of power production plants a good maintenance plan can double the lifetime of core equipment [13]. One of the first work concerned with both problems is presented in [14]. It presents a production planning for one machine and takes into account the outage time and the production loss caused by the preventive maintenance. This paper proved for one of the first times that it is possible to improve profits by implementing a great maintenance planning in coordination with the production, but it is based on a theoretical case rather than a complex industrial system. In the field of electrical production domain the main difficulty of the problems is that the production functions are nonlinear, but some linearization methods for these functions can be applied [15]. In [16] an approach using a two stages optimization model for solving the GMSP in a nuclear plant is presented. The first stage generates a flexible planning for all the maintenance tasks. This flexible planning is based on maintenance costs, time, emergency levels of maintenance and machine reliability. The second stage determines the optimal place for each maintenance in the schedule. The main difficulty of this problem is that the production functions of the nuclear turbines are nonlinear. Thus the problem is solved using a genetic algorithm, which provides good solutions fast, but has not proof of optimality. In [17] the impact of operations decision and maintenance planning on a power plant are discussed. For a maintenance task there are a lot of factors to take into account like time, costs, startup and shutdown times that are not negligible. There is also workforce costs that can be different depending of the weekday or even the season (given summer vacations, for example). Other technical factors can be operational costs on generators and optimal operating range. All these factors are often too simplified or even neglected. A model including that many factors could have a great precision and be the closest to the reality. With such a model, the maintenance schedule would be truly optimal, but modelling so many factors increases the size of the problem and it is nearly impossible to solve. A survey is presented in [18], with many test cases related to hydropower. The main conclusion of this work is that it is difficult to model the problems to stick to reality while being time efficient in the resolution. In the proposed survey, important parts of the problems neglects or over simplify some important parts of the problem like the generator's nonlinear production functions or the water level of reservoirs [19, 20, 21, 22, 23, 24, 25, 26]. In [19] the authors presents a MIP to solve the GMSP for a mixed production system (it is composed by coal, nuclear and hydropower plants). This MIP gives a maintenance planning for the generators, a production schedule and takes decisions about the transport and mining of raw materials for coal plants. Thus there is no production demand to take into account and the hydropower generator's production functions are linear. The hydropower system is simply a variable defining the availability of power that can be generated at each period. Consequently, the hydropower decision is not clearly impacting the next period with a water conservation constraint, for example. In [20] an approach to compute the optimal load of water during an outage of one or more generators is discussed. The hydropower problem is formulated as

a linear model. This linearization of production functions simplifies the problem. After solving the hydropower linear problem, a simulation in which each maintenance tasks combinations is processed and the best one is selected to be the optimal schedule. This kind of model is hard to scale to a bigger installation. An approach based on fuzzy logic is presented in [21], but it does not take into account some important hydropower characteristics such as the water level in reservoir. In [22] an ant colony algorithm is defined to solve the GMSP for a system composed with five plants, but this study is not interested by the nonlinear characteristic of the problem and it is a long-term deterministic approach. Another study [23] presents an oversimplified approach that is suppose that the power produced by a generator is constant, therefore the water head is neglected. In [24] authors use Bender's decomposition to find an optimal schedule. This approach does not take individually the power production of the plants, and only the reservoir level is considered. The Bender's decomposition master problem objective function only minimize the water usage. In [25] the modelisation of GMSP is modified by adding a time window for each outage. This modification improves the maintenance reality challenges. Also, some constraints are modelled to optimize the number of workers for each scheduled maintenance. Again the nonlinearity of the power functions is not considered. This is proving that adding a time window for outages duration is improving the results and that it generates a schedule that is closest to the reality. In [26] a short-term model for maintenance in a wind turbine installation is shown. Wind turbines are really close to hydropower plants because of the nonlinearity of the generator's production function and direct link of forecasts with future production. A metaheuristic solve their problem and showed that making a short-term schedule for maintenance improves the production of a wind turbine installation. The production functions are too simplified by just using the forecast to define the production. According to [1], simplifying the GMSP by neglecting the nonlinear aspect of the production function can lead to an over estimation of the power production. This can lead to suboptimal maintenance schedules. Thus, the hydropower production problem is already hard to solve, even more if it is a problem composed with many plants on the same water stream. It is not trivial to consider hydrologic inflows and all the difficulties of a full maintenance scheduling problem. In this context, it can be interesting to make some mathematical simplifications if this can lead virtually to find a good solution in a suitable time.

Another part of the literature represents more precisely the key elements of the hydropower problem. In [27] a genetic algorithm is presented to solve the GMSP using continuous variables to represent the starting time and time windows of the maintenance. The power function is modeled with an analytical function. Therefore, this paper neglects the nonlinear effect of the generator numbers change over the final power production. A chance constraint model that takes into account the stochastic inflows is presented in [28]. The power demand and the nonlinear variable of hydropower optimization are taken into account. For simplification, the deterministic equivalent of the problem is solved and nonlinear effect of the generator numbers change over the final power production is neglected. In [29] a multistage stochastic model is described where maintenance scheduling decision are taken at the first stage and hydropower operational decision are taken over the next stages. Also, a Bender's decomposition algorithm solve their model. The stochastic natural inflows are considered with a scenario tree and a stochastic selling price of power is emulated using Markov chains in a liberal electricity market state.

All the cited papers are interesting but most of them present simplifications that lead to unrealistic models in practice. Also the papers presenting models that are not oversimplified are using a single power production function for each power plants. Thus, the model presented in this paper uses power production functions for a certain number of active generators, instead of individual power production functions. For example, if the power plant has 5 generators and engineers decide that 2 maintenances can be performed at the same time, the model has too chose from 3 power production functions, one for 3 active generators, one for 4 active generators and one for 5 active generators. There is a slight difference in the power production when turning off one or another generator but it is not significative for the purpose of maintenance planification. In this case, a precise short-term optimization plan is already used for daily operation and only the small power loss of planning outages has to be modelled for planification purposes. This paper is based on previous works that can be found in [1, 2]. The

first paper is about a very precise model proposition to solve the GMSP but it is not fast enough for daily operation planning. The second paper is an implementation of Bender's decomposition to accelerate the solving time and consider uncertain inflows in the model but it proves that Bender's decomposition has a better solving time with a lot of inflows scenarios, but it has almost no benefits to plan a lot of maintenance's tasks. With today's scenario trees technology, problems can be solved with a decent amount of scenarios and a precise liability so it should be better to have an algorithm that scales better with maintenances number than scenario number. These two models provide suitable maintenance plans, but the solving time is high. In this paper a reformulation of the problem is proposed by adding a innovative way to evaluate only one power function per power plant. By doing this, it is possible to dramatically reduce the number of constraints and to step up the solving time. With a reduced solving time, it is possible to increase the amount of maintenances to schedule or even solve the model more often to take better decisions over a rolling horizon. *The paper is organized as follows.* Section 2 presents the mathematical model. Numerical results are presented in Section 3 and final remarks are raised in Section 4.

2 The generator maintenance scheduling problem

The problem that is being modeled in this section is the GMSP. It is known to contain a lot of constraints and integer variables. It is complicated to solve a nonlinear integer program of this size. That's why it is formulated as a Mixed Integer Linear Program (MILP). To do so, the original nonlinear hydropower production function are reformulated as a set of hyperplanes.

2.1 Approximation of the hydropower production functions using hyperplanes

The unit production function of each plant is a polynomial function of the form :

$$f(x, y) = p_{00} + p_{10}x + p_{01}y + p_{20}x^2 + p_{11}xy + p_{02}y^2 + p_{30}x^3 + p_{21}x^2y + p_{12}xy^2 + p_{40}x^4 + p_{31}x^3y + p_{22}x^2y^2 \quad (1)$$

with x the discharged water in m^2/s and y the reservoir volume in m . Each $P_{ij}, i \in \{0, 1, \dots, 4\}, j \in \{0, 1, 2\}$ is a coefficient. This polynomial function can be used to model one generator production function or a complete plant with a combination of active generators. In the case of maintenance scheduling, it is mandatory to model the difference between the production function of a plant with n active generators. The most reliable mathematical model to do this is to use a nonlinear fitting of the unit production function per generator and the production function of the plant is the sum of all generator's unit production functions. Therefore, in practice, depending of the plant type, it is difficult to decide the amount of discharged water per generator because the valve have no reliable fine tuning settings due to high water pressure. Taking this into account, in the hydropower optimization models, a production function per plant is used. In the model that use a more precise formulation, a production function per plant and per generator combination is used. While using this formulation it is possible to note the production difference if a generator or more is turned off. It is possible to solve this nonlinear unit load balancing problem, but adding a maintenance schedule to it for many interconnected plants would be very hard to solve. Thus to make this problem feasible for a large-scale installation of multiple plants, even with a single production function instead of one production function per generator, the presented problem is linearized because solving a MILP with today's mathematical tools, is faster than solving a nonlinear integer problem, even if the linear problem is larger. To linearize the problem each production function is represented with a set of hyperplanes. Figure 1 shows how the hyperplanes represent the nonlinear function. An algorithm presented by [31] is used to find a set of hyperplanes that fit the production function of the power plant. This algorithm chooses a point on the production function's definition domain and calculate a tangent plane. Then it searches for the point where there is the greatest difference between the production function and the plane created and it generates another plane. The algorithm stops when all points of the function have a difference with the hyperplane set lower than ϵ .

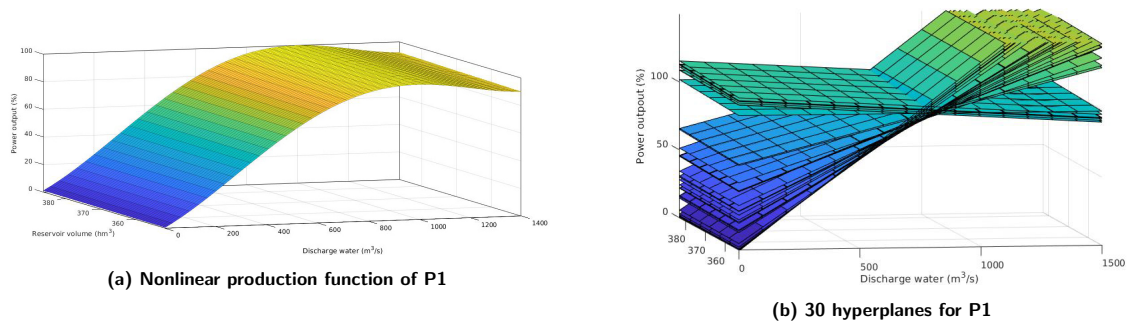


Figure 1: Comparison between the 2 production function models

Once the hyperplanes set are defined for all powerplants, the model can be formulated as a MILP. This formulation comes with the advantage to be linear but trading one production function with a set of hyperplanes increases the number of constraints. However, nonlinear mixed integer problem are really hard to solve and transforming this nonlinear problem in a linear problem increases the performances even if the problem is larger. The number of hyperplanes required to approximate the hydropower production function is directly linked to the ϵ selected in the algorithm. The smaller ϵ is, the larger the number of hyperplanes will be. For instance, if a single function is substituted by 30 hyperplanes, the model has 30 more constraints to take into account. This is not a big difference for a single operation, but in a case where there are 30 periods to evaluate for 5 different power plants for many different combinations of generators it is a huge increase of the problem size. Moreover, the presented model is a stochastic MILP so all of these production functions constraints are again multiplied by the number of scenarios. With all these hyperplanes to evaluate, the complexity of the problem is undeniably linked to the number of hyperplanes selected for each generator combinations. Solutions to reduce the size of the problem with a minimal loss of viability are discussed in this paper. Table 1 shows the different number of constraints linked with the number of hyperplanes per functions.

Table 1: Comparison between number of hyperplane versus total number of constraints

Number of hyperplane	Number of constraint
5	110,864
10	182,864
20	326,864
30	470,864
40	614,864

Now that the formulation techniques used to linearize the problem are presented, it is possible to discuss how the number of constraints have been reduced.

2.2 Heuristic proposed to reduce the number of hyperplanes

Given the size of this problem is directly linked to the precision of the hyperplanes modelling, a methodology is proposed to reduce the number of hyperplanes for many periods of the problem. In some periods it is impossible to schedule maintenances because of industrial constraints or simply because there are no maintenances to do during a specific period. In those periods, it is mandatory to choose the maximum number of generators and the production function of the plant is always the one with the highest number of active generators. Then, for the periods when generator maintenances can be planned, a heuristic is used to evaluate only one production function.

Instead of using one set of n hyperplanes per generator combination fore each time step and power plant, a heuristic is a proposed. The heuristic is made to use a \mathcal{R} value to reduce or raise the production

function to estimate the production with more or less active generators. For instance, if a powerplant has 5 generators and the maximum outages is set to 3, this powerplant has 3 production functions (3, 4 or 5 active generators). In this case the production function with 4 active generators is always used and \mathcal{R} value is added to raise or lower the final power of this function if there is one more or less generator. This \mathcal{R} value is found by using a mean square algorithm to find the best match between the production function and all the other production functions of the power plant. In this context, for all the power plants and for all the generator combinations, a unique \mathcal{R} is found to estimate the power output of the plant. This method is less accurate than the one with a set of hyperplanes for all combinations but it models the difference of power output if a generator is stopped or restarted. In the GMSP, the main goal is to formulate the loss of power due to the outages and with this technique, this loss of power is represented without increasing the size of the problem. Figure 2 presents the difference between the production function estimated by the hyperplane modelisation versus the heuristic. This Figure shows only the comparison with all active generators and 2 turned off generators because they are both estimated with the production function of the plant with one turned off generator. This data comes from P3 (Figure 3 presents the system used to retrieve the data.). It is possible to remark that in both cases the estimation is accurate. There is a small loss of precision but the curve tendency is preserved. It shows that it is possible to estimate efficiency loss by using only one function.

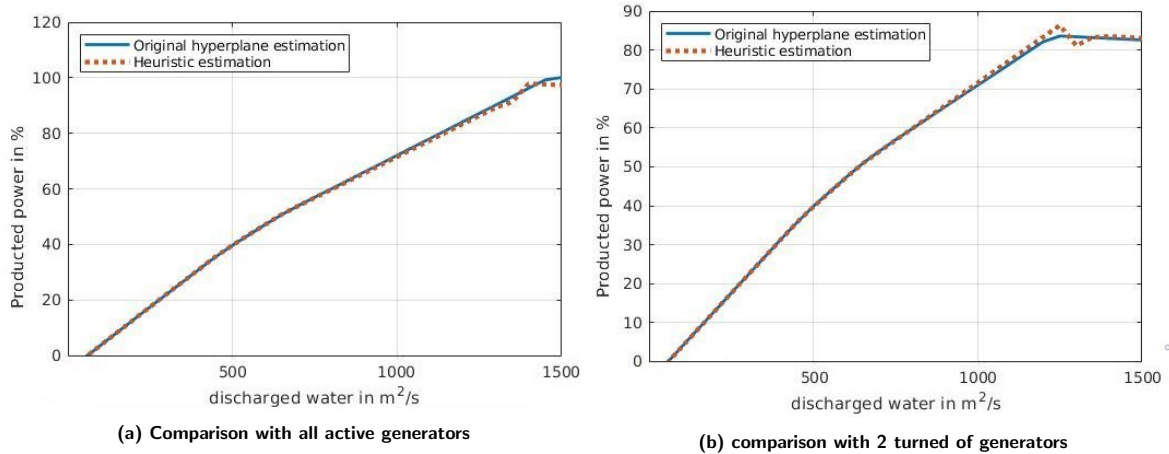


Figure 2: Comparison between the heuristic vs hyperplanes of the estimated production function

2.3 Problem formulation

This problem has random inflows, thus, a stochastic problem has to be solved and a two-stage stochastic program is modelled to account for the uncertain inflows. The hydropower decisions are made at the first stage, then the maintenance planning is defined over the next stage. This problem uses a 30-day horizon where decision has to be made for each day for the hydropower problem. A maintenance planning has to be scheduled over the whole horizon. This maintenance planing is composed of maintenance task for generators that need one. Each maintenance task has a time window to be processed and a duration. The model has to choose the best place for the maintenance in the time window. Time windows are set by engineers with the help of monitoring tools and experience. This stochastic model uses a scenario fan of 40 inflows scenarios coming from historical data. A scenario fan is a set of individuals scenarios. The mathematical model is presented below.

Sets and parameters

$\mathcal{I} = \{1, 2, 3, 4, 5\}$	The set of power plants;
$\mathcal{T} = \{1, 2, \dots, 30\}$	The set of periods;
$\mathcal{M} = \{1, 2, \dots, m\}$	The set of maintenance tasks;
$\mathcal{K}(i, t) = \{1, 2, \dots, n\}, i \in \mathcal{I}, t \in \mathcal{T}$	The set of active generators at the period t and the powerplant i ;
$\Omega = \{1, 2, \dots, 40\}$	The set of inflows scenarios;
$\mathcal{H}_i \forall i \in \mathcal{I}$	The set of hyperplanes representing the estimated production function of the power plant i ;
$\mathcal{R}_{i,k} \forall i \in \mathcal{I}, k \in \mathcal{K}$	The set of constants numbers for correcting the estimated power at the power plant i while k turbines are actives;
$B_t^+ \forall t \in \mathcal{T}$	The price of electricity sold on the electricity market at the period t ;
$B_t^- \forall t \in \mathcal{T}$	The price of electricity purchase on the electricity market at the period t .

Decision variables

$y_{mt} =$	$\begin{cases} 1 & \text{if the outage } m \in \mathcal{M} \text{ starts at the period } t \in \mathcal{T}, \\ 0 & \text{otherwise;} \end{cases}$
$z_{itk} =$	$\begin{cases} 1 & \text{if } k \in \mathcal{K}(i, t) \text{ generators } i \in \mathcal{I} \text{ are actives during the period } t \in \mathcal{T}, \\ 0 & \text{otherwise;} \end{cases}$
$v_{it\omega} \forall i \in \mathcal{I}, t \in \mathcal{T}, \omega \in \Omega$	The spillway flow at the plant i at the period t for the scenario ω ;
$u_{it\omega} \forall i \in \mathcal{I}, t \in \mathcal{T}, \omega \in \Omega$	The discharge flow at the plant i at the period t for the scenario ω ;
$s_{it\omega} \forall i \in \mathcal{I}, t \in \mathcal{T}, \omega \in \Omega$	The upstream elevation at the plant i at the period t for the scenario ω this variable is used to calculate the reservoir volume;
$q_{t\omega}^+ \forall t \in \mathcal{T}, \omega \in \Omega$	The electricity sold a the period t for the scenario ω ;
$q_{t\omega}^- \forall t \in \mathcal{T}, \omega \in \Omega$	The electricity bought a the period t for the scenario ω ;
$n_{it} \forall i \in \mathcal{I}, t \in \mathcal{T}$	The number of maintenance for the plant i at the period t .

The problem is written :

$$\text{maximize}_{q^+, q^-, u, v, s, y, z} \sum_{\omega \in \Omega} \left(\frac{1}{|\Omega|} \left(\sum_{t \in \mathcal{T}} B_t^+ q_{t\omega}^+ - \sum_{t \in \mathcal{T}} B_t^- q_{t\omega}^- \right) \right) \quad (2)$$

Subject to :

$$\sum_{t \in \mathcal{T}} y_{mt} = 1, \quad \forall m \in \mathcal{M}, \quad (3)$$

$$\sum_{\substack{m \in \mathcal{M}(i) \\ t' \in \mathcal{T}(m) \cap [t-Dm+1, t]}} y_{mt'} = n_{it}, \quad \forall i \in \mathcal{I} \forall t \in \mathcal{T}, \quad (4)$$

$$n_{it} + \sum_{k \in \mathcal{K}(i, t)} kz_{itk} = G_{it}, \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T}, \quad (5)$$

$$\sum_{k \in \mathcal{K}(i, t)} kz_{itk} = 1, \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T}, \quad (6)$$

$$n_{it} \geq 0, \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T}, \quad (7)$$

$$v_{it\omega} \geq 0, \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T}, \forall \omega \in \Omega, \quad (8)$$

$$0 \leq u_{it\omega} \leq U_{it}^+, \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T}, \forall \omega \in \Omega, \quad (9)$$

$$0 \leq s_{it\omega} \leq S_{it}^+, \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T}, \forall \omega \in \Omega, \quad (10)$$

$$0 \leq q_{t\omega}^+ \leq W_t^+, \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T}, \forall \omega \in \Omega, \quad (11)$$

$$0 \leq q_{t\omega}^- \leq W_t^+, \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega, \quad (12)$$

$$p_{it\omega} = \sum_{k \in \mathcal{K}(i, t)} p_{itk}\omega, \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T}, \forall \omega \in \Omega, \quad (13)$$

$$\sum_{i \in \mathcal{I}} p_{it\omega} + q_{t\omega}^- = A + q_{t\omega}^+, \quad \forall t \in \mathcal{T}, \forall \omega \in \Omega, \quad (14)$$

$$s_{it\omega} - s_{i(t-1)\omega} = (\sigma_{it\omega} + \sum_{g \in \mathcal{U}(i)} (u_{gt\omega} + v_{gt\omega}) - u_{it\omega} - v_{it\omega} \times F), \quad \forall i \in \mathcal{I}, \forall t \in \mathcal{T}, \forall \omega \in \Omega, \quad (15)$$

$$\begin{aligned}
p_{itk\omega} &\leq \beta_h^0 + \beta_h^u u_{it\omega} + \beta_h^s s_{it\omega} && \forall i \in \mathcal{I}, \forall t \in \mathcal{T}, \forall \omega \in \Omega, \\
&+ \sum_{k \in \mathcal{K}(i,t)} k z_{itk} * \mathcal{R}_{ik} && \forall k \in \mathcal{K}(i,t), \forall h \in \mathcal{H}(i). \quad (16)
\end{aligned}$$

The objective function Equation (2) maximizes the profit by subtracting electricity bought to electricity sales. In this case, all the maintenance have the same hourly rate every day. Thus, there is no incidence to add it or not in the objective function. Constraints (3)–(7) are the constraints relative to the maintenance problem. Equation (3) verifies if all the outages are scheduled for the optimization horizon. Constraints (4) compute the amount of maintenance n_{it} per plant and per period. Constraints (5) is link the number of outages n_{it} to the number of active generators. Constraints (6) checks if only one combination of generators is active for each period. Constraints (7) ensure the variables n_{it} are non-negative. Constraints (8)–(14) are the constraints relative to the hydropower problem. Constraints (8)–(12) are the hydropower bounds. Constraint (15) are the water conservation constraint. Set $\mathcal{U}(i)$ is the set of power plants that are upstream of the plant $i \in \mathcal{I}$, the parameter F is a multiplication factor to change m^3/s inflows into a daily reservoir volume in hm^3/j . Constraints (16) estimate the power produced by the plant using the number of active generators defined by the variable z_{itk} . The coefficients β_h are the coefficients of the hyperplane h . Constraints (13) sums the production of each plant for the scenario ω . The constraints (14) verify that the production and purchases of electricity are enough to power the aluminum production (A_t) and sell the surplus.

3 Computational experiments

This section presents the numerical results conducted to validate the proposed approach. The results obtained by this study use data from a real case located in Québec (Canada). It is the Saguenay-Lac Saint-Jean's hydropower system. This system is composed of 4 power plants connected in series. Two power plants are located of the Lac Saint-Jean's reservoir, and the 2 others are downstream. The Lac St Jean's reservoir elevation is managed by strict rules because of recreative purposes during summer, and constraints are added to cover these seasonal rules. All the power plants have different numbers of generators (between 5 and 17). With this number of generators, it is obviously difficult to solve (in a reasonable space and time complexity) a model using the unit production functions of all generators. It is also hard to solve a model which represents all the generator's combinations. Two power plants are run-of-the-river power plants (P2 and P4) and the 2 other plants have reservoirs. The Figure 3 present a global view of the Saguenay lac Saint-Jean's hydropower system.

To make a fair comparison, the two models were tested with same input parameters and then all the operational decision given by both models are tested with the real case power functions. This way all the operational decisions are tested with the same function to have a comparable power output and verify the difference between the production of the two different solutions. The comparison of the evolution of production is made with real nonlinear functions. To do so, we used all the decisions taken by both models and we simulated it with the original nonlinear modelisation of power function of all plants. The first model (Model A) tested is the model presented in [1]. This model proposes a solution to the GMSP using a stochastic MILP with a precise definition of all combinations of production functions. This model is used to make a comparison of the classical method used to solve this problem and the formulation presentend in this paper. The second model (Model B) is the model proposed in this paper which is reducing the number of constraint by using an heuristic to estimate the power loss or gain by using more or less generator during a maintenance. In summary, the Model B uses only one function to represent the nonlinear variation due to the outage of one or more generators while the Model A uses one function for each combination of generators. The Model B performs better in computation time with a minimal loss in power production estimation. Model B obtains result in a faster computing time so it can be run often if it is decided to run it in rolling horizon for future works. The estimated power production loss from the Model B versus the Model A is not considered significant for this work because the purpose of this algorithm is to find a good maintenance plan and

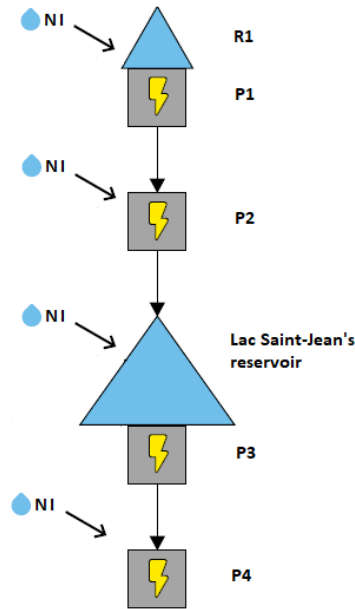


Figure 3: Saguenay lac Saint-Jean's hydropower system, where NI are Natural Inflows, P represent a power plant and R a reservoir

not to estimate the power function to be the closest to reality. To test the Model B, we solved the two stages stochastic program with GLPK [30]. In this paper, one detailed instance is presented. This instance of 16 outages over the 30-day horizon. Table 2 shows all the maintenance tasks to schedule. The location column is the power plant index. For this use case, there are 16 maintenance tasks to schedule. The inflow uncertainty is represented by a 40 historical inflows scenario fan. The instance that is presented more precisely is the instance number 15 in the Figure 9 and Figure 10. This instance is chosen because both models presented a different maintenance scheme and the power gap between both models is average versus all the other cases. Another instance could have been chosen, but it is more reliable to present an average case rather than an extreme one.

Table 2: Example of a maintenance plan to schedule

Maintenance number	Duration (in periods)	Minimal starting period	Maximal starting period	Location
1	4	1	3	0
2	5	2	4	0
3	7	6	8	0
4	5	8	10	0
5	6	0	2	1
6	4	1	3	1
7	6	7	9	1
8	4	7	9	1
9	3	0	2	2
10	6	7	9	2
11	3	2	4	2
12	4	6	8	2
13	2	11	13	3
14	4	9	11	3
15	5	3	5	3
16	5	0	2	3

All the tested cases are performed on a laptop computer with an Intel Core i7 Processor and 16 GB of RAM.

The results for 2 of the 4 power plants are presented in details, more precisely “P1” and “P3”. The 2 other plants are omitted from the results since they are run-of-the-river plants. Figure 4 and Figure 5 shows the evolution of estimated productions versus the discharged water of both models over the time horizon. To respect data confidentiality of the company, all values are replaced by percentages and the 100% value is the maximum bound of each dataset. For example, if the reservoir level is 0%, it is at its minimal bound and it is impossible to discharge more for the period. It is possible to notice in the Figure 6 and Figure 7 that there is a slight difference between both reservoir levels but the curve tendency is still the same between the two models. This proves that the Model B has the same behavior than the most precise model. Also the new model uses less water in the first periods than the previously presented model. In practice, this can be usefull for engineers and with this tendency they can easily avoid to run out of water too fast in case of low natural inflows.

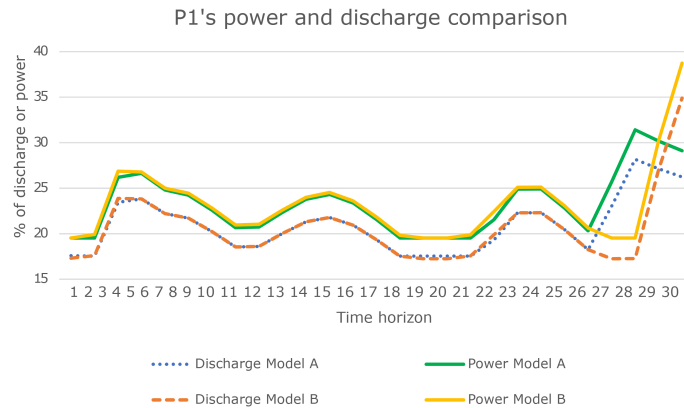


Figure 4: “P1” power and discharge comparison between both models for one instance

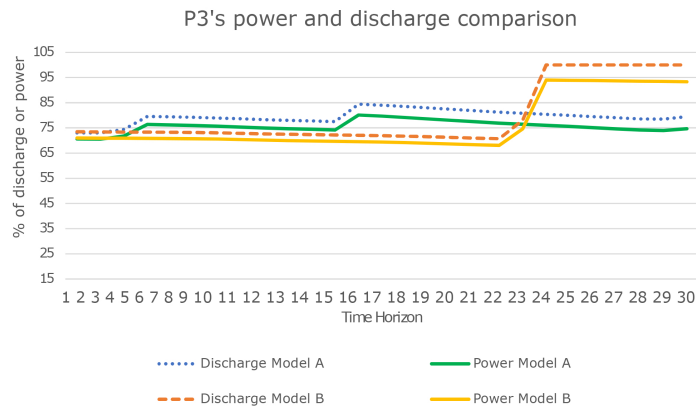


Figure 5: “P3” power and discharge comparison between both models for one instance

Next, Figure 8 presents both models generated planning. As we can see, they have very small differences. The only 2 differences between both models are on that Mt3 and Mt11 are scheduled with a difference of 1 period. both models propose almost the same scheduling but with a great computation time difference.

Finally, the solving time of this model is significantly better (approximately 5 times better) than the previously developed model. Indeed, for the 16 instances, the mean solving time is approximately 30 seconds for this model, for the previous model, the solving time mean is approximately 130 seconds. This solving time gain can be used to use more scenarios to try a better stochastic approach, for running the model often on a rolling horizon planning or even planning more maintenance tasks. These time changes are more significant than the small power loss due to our simplified model. Also, the original

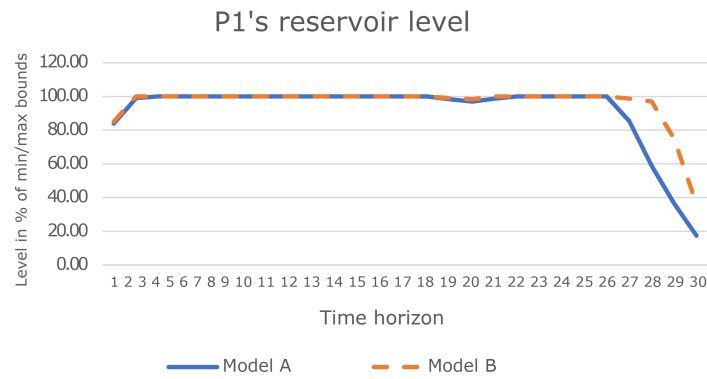


Figure 6: P1's reservoir level comparison between both models

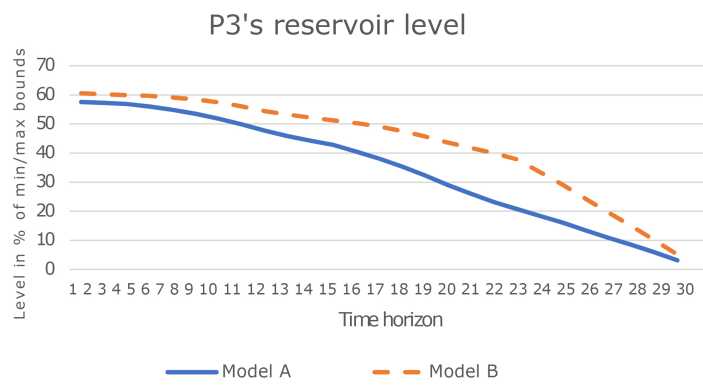


Figure 7: P3 plant's reservoir level comparison between both models

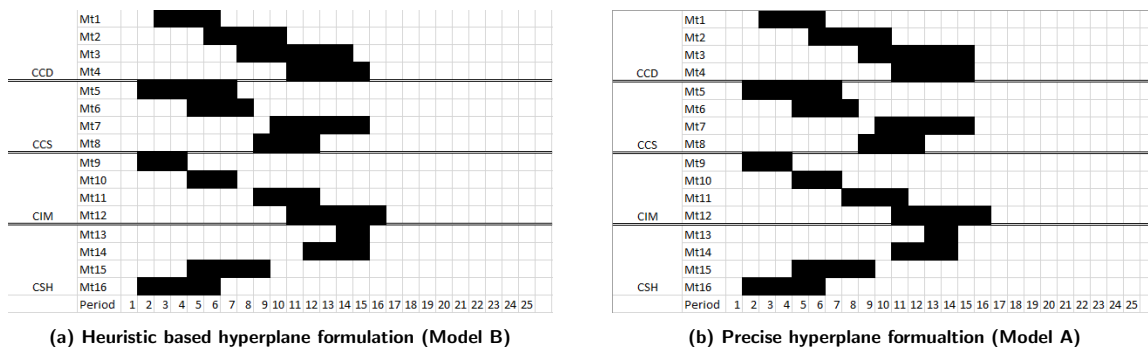


Figure 8: Comparison between the 2 models' generated plannings

model is running more than 30 minutes in some cases where there is no solution (over constrained). The Model B stops quickly if there is no feasible solution. Figure 9 shows the difference of solving time for all tested instances.

The Figure 10 shows the total energy produced estimated by both models in comparison. Again, real numbers are not shown but percentages are used to show the difference between both models. The 100% on the energy scale is the maximal energy that can be produced if all plants are at their best yield and all reservoirs are always full. As we can see, the new estimation of the production function has a small energy loss overall but it is at least five times faster than the first version of the model. Again this last Figure shows that the production tendency is similar between the two models and infers that the Model B presented in this paper finds good results in a significant lower time.

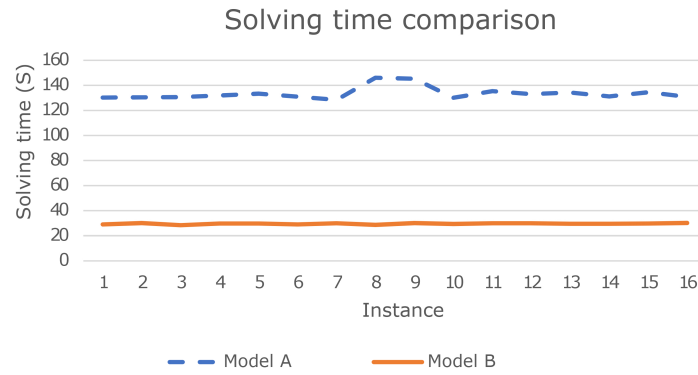


Figure 9: comparison of the two models solving time

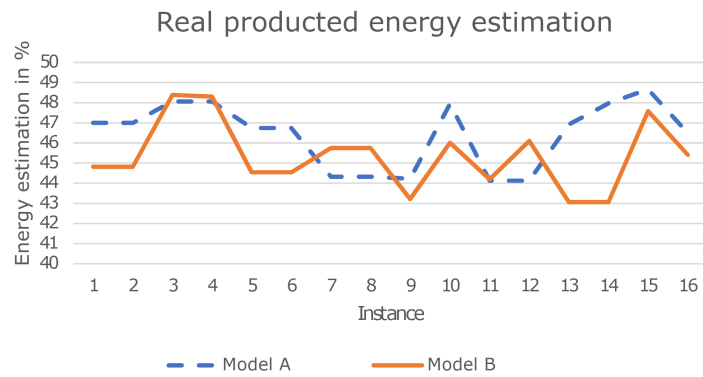


Figure 10: Comparison of the two models' productions for all 16 tested instances

4 Conclusion and future works

With this new model, the generator maintenance scheduling problem was simplified to generate a fast solution with less precision about operational decision but with a significant gain of solving time. This little loss of energy is not considered critical for this problem because the role of the GMSP is to find a maintenance schedule in coordination with operational decisions but it has to be used, in real cases, in coordination with a short-term optimization model for the operational decisions. Indeed, this GMSP will schedule the outages of generators, and then a short-term, very fine-tuned algorithm is used to define the operational decisions with the number of generators given by the GMSP. Moreover, this model computes good maintenance decisions and the operators can use it directly. This model is running fast and gives acceptable decisions but it is still improvable. In our future works, we will focus on using a decomposition algorithm to accelerate the solving time and to add some operational constraints like splitting some maintenances if there is a big inflow forecasted or adding workforce availability in the modelisation. Also it is planned to work on re-optimization of the given solutions to run this model in a rolling horizon fashion. If there is a big change in forecasts or a new emergency maintenance to schedule, the algorithm will be able to take a good decision. Finally, to improve the model reliability the maintenance definition can be improved. For instance, we can refine the direct maintenance costs with material costs, different workforce hourly rates for week and weekend or moving costs between the plants for example.

References

- [1] J. A. Rodríguez, M. F. Anjos, P. Côté, and G. Desaulniers, MILP Formulations for Generator Maintenance Scheduling in Hydropower Systems, *IEEE Trans. Power Syst.*, 33(6):6171–6180, Nov. 2018,

- doi: 10.1109/TPWRS.2018.2833061.
- [2] J. A. Rodríguez, M. F. Anjos, P. Côté, and G. Desaulniers, Accelerating Benders decomposition for short-term hydropower maintenance scheduling, *European Journal of Operational Research*, 289(1):240–253, Feb. 2021, doi: 10.1016/j.ejor.2020.06.041.
 - [3] D. Schwanenber, F. M. Fan, S. Naumann, J. I. Kuwajima, R. A. Montero, and A. Assis dos Reis, Short-Term Reservoir Optimization for Flood Mitigation under Meteorological and Hydrological Forecast Uncertainty: Application to the Três Marias Reservoir in Brazil, *Water Resour Manage*, 29(5):1635–1651, Mar. 2015, doi: 10.1007/s11269-014-0899-1.
 - [4] S. Séguin, P. Côté, and C. Audet, Self-Scheduling Short-Term Unit Commitment and Loading Problem, *IEEE Trans. Power Syst.*, 31(1):133–142, Jan. 2016, doi: 10.1109/TPWRS.2014.2383911.
 - [5] S. Séguin, S.-E. Fleten, P. Côté, A. Pichler, and C. Audet, Stochastic short-term hydropower planning with inflow scenario trees, *European Journal of Operational Research*, 259(3):1156–1168, Jun. 2017, doi: 10.1016/j.ejor.2016.11.028.
 - [6] A. Marchand, M. Gendreau, M. Blais, and J. Guidi, Optimized operating rules for short-term hydropower planning in a stochastic environment, *Comput Manag Sci*, 16(3):501–519, Jul. 2019, doi: 10.1007/s10287-019-00348-2.
 - [7] M. M. Belsnes, O. Wolfgang, T. Follestad, and E. K. Aasgård, Applying successive linear programming for stochastic short-term hydropower optimization, *Electric Power Systems Research*, 130:167–180, Jan. 2016, doi: 10.1016/j.epsr.2015.08.020.
 - [10] D. Nyman and J. Levitt, Maintenance planning, scheduling, and coordination. Industrial Press Inc., 2001.
 - [11] P. Lyonnet, Maintenance planning: Methods and mathematics. Springer Science & Business Media, 2013.
 - [12] D. Palmer, Maintenance planning and scheduling handbook. McGraw-Hill New York, 2006.
 - [13] Fichtner, Hydroelectric Power: A Guide for Developers and Investors., International Finance Corporation, 2015.
 - [14] H. L. Lee and M. J. Rosenblatt, A Production and Maintenance Planning Model with Restoration Cost Dependent on Detection Delay, *IIE Transactions*, 21(4):368–375, Dec. 1989, doi: 10.1080/07408178908966243.
 - [15] E. EDOM, M.F. Anjos, C. D’Ambrosio, W. van Ackooij, P. Côté and S. Séguin, On the impact of the power production function approximation on hydropower maintenance scheduling, Les Cahiers du GERAD G-2020-22, GERAD, HEC Montréal, Canada, Technical report, Apr. 2020.
 - [16] C. M. F. Lapa, C. M. N. A. Pereira, and M. P. de Barros, A model for preventive maintenance planning by genetic algorithms based in cost and reliability, *Reliability Engineering & System Safety*, 91(2):233–240, Feb. 2006, doi: 10.1016/j.res.2005.01.004.
 - [17] I. B. Stoenescu, S. Costinas, and G. M. Deaconu, Assessment of Hydropower Plants Energy Production Cost Influenced by Operational Decisions and Control Strategy, in 2019 22nd International Conference on Control Systems and Computer Science (CSCS), Bucharest, Romania, May 2019, pp. 347–352. doi: 10.1109/CSCS.2019.00062.
 - [18] A. Froger, M. Gendreau, J. E. Mendoza, É. Pinson, and L.-M. Rousseau, Maintenance scheduling in the electricity industry: A literature review, *European Journal of Operational Research*, 251(3):695–706, Jun. 2016, doi: 10.1016/j.ejor.2015.08.045.
 - [19] D. Chattopadhyay, K. Bhattacharya, and J. Parikh, A systems approach to least-cost maintenance scheduling for an interconnected power system, *IEEE Trans. Power Syst.*, 10(4):2002–2007, Nov. 1995, doi: 10.1109/59.476069.
 - [20] R. Sunusi and K. Indriawati, Study of dam operating strategy for optimal water management during preventive maintenance period on PLTA PT. Vale Indonesia, Surabaya, Indonesia, 2019, p. 030007. doi: 10.1063/1.5095312.
 - [21] Y. Feng, P. Li, and H. Wang, Hydro-thermal generator maintenance scheduling accommodating both randomness and fuzziness, in 2011 4th International Conference on Electric Utility Deregulation and Restructuring and Power Technologies (DRPT), Weihai, China, Jul. 2011, pp. 734–741. doi: 10.1109/DRPT.2011.5993989.
 - [22] W. K. Foong, A. R. Simpson, H. R. Maier, and S. Stolp, Ant colony optimization for power plant maintenance scheduling optimization—a five-station hydropower system, *Ann Oper Res*, 159(1):433–450, Mar. 2008, doi: 10.1007/s10479-007-0277-y.
 - [23] S. Perez-Canto and J. C. Rubio-Romero, A model for the preventive maintenance scheduling of power plants including wind farms, *Reliability Engineering & System Safety*, 119:67–75, Nov. 2013, doi: 10.1016/j.res.2013.04.005.

- [24] I. Kuzle, H. Pandžić, and M. Brezovec, Hydro generating units maintenance scheduling using Benders decomposition, *Tehnički vjesnik*, 17(2):145–152, 2010.
- [25] U. E. Ekpenyong, J. Zhang, and X. Xia, An improved robust model for generator maintenance scheduling, *Electric Power Systems Research*, 92:29–36, Nov. 2012, doi: 10.1016/j.epsr.2012.03.016.
- [26] C. Stock-Williams and S. K. Swamy, Automated daily maintenance planning for offshore wind farms, *Renewable Energy*, 133:1393–1403, Apr. 2019, doi: 10.1016/j.renene.2018.08.112.
- [27] L. Guedes, D. Vieira, A. Lisboa, and R. Saldanha, A continuous compact model for cascaded hydro-power generation and preventive maintenance scheduling, *International Journal of Electrical Power & Energy Systems*, 73:702–710, 2015.
- [28] X. Ge, S. Xia, and X. Su, Mid-term integrated generation and maintenance scheduling for wind-hydro-thermal systems, *Int Trans Electr Energ Syst*, 28(5):e2528, May 2018, doi: 10.1002/etep.2528.
- [29] A. Helseth, M. Fodstad, and B. Mo, Optimal Hydropower Maintenance Scheduling in Liberalized Markets, *IEEE Trans. Power Syst.*, 33(6):6989–6998, Nov. 2018, doi: 10.1109/TPWRS.2018.2840043.
- [30] GNU Linear Programming Kit, Version 4.65, <http://www.gnu.org/software/glpk/>
- [31] P. Côté & R. Arsenault (2019). Efficient implementation of sampling stochastic dynamic programming algorithm for multireservoir management in the hydropower sector. *Journal of Water Resources Planning and Management*, 145(4):05019005.