Les Cahiers du GERAD ISSN: 0711–2440

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G–2024–54

Septembre 2024

La publication de ces rapports de recherche est rendue possible grâce au soutien de HEC Montréal, Polytechnique Montréal, Université McGill, Université du Québec à Montréal, ainsi que du Fonds de recherche du Québec – Nature et technologies.

Dépôt légal – Bibliothèque et Archives nationales du Québec, 2024
– Bibliothèque et Archives Canada, 2024

The publication of these research reports is made possible thanks to the support of HEC Montréal, Polytechnique Montréal, McGill University, Université du Québec à Montréal, as well as the Fonds de recherche du Québec – Nature et technologies.

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Optimal timing of major projects when time-to-build and investment cost are uncertain

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Septembre 2024

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Abstract : Major projects often deviate from their expected budget, schedule, and benefits. Existing techniques to improve the forecasted outcome, such as the Reference Class Forecasting technique, ignore any flexibility in the project timing. This paper acknowledges this flexibility and proposes a real options model to optimise the project timing and value when the project investment cost, the construction schedule, and the output value are uncertain. We illustrate the approach with thin and fat upper tails distributions of cost and schedule overruns. Finally, we show how practitioners can implement the proposed approach using empirical distributions of costs and schedules from comparable prior projects.

Keywords: Real options, major projects, delays, cost overruns, uncertainty, reference class forecasting

Résumé : Les grands projets s'écartent souvent de leur budget, de leur calendrier et de leurs bénéfices attendus. Les techniques existantes pour améliorer les prévisions de résultats, telles que la technique de prévision par référence à une classe, ignorent toute flexibilité dans le calendrier du projet. Cet article prend en considération cette flexibilité et propose un modèle d'options réelles pour optimiser le calendrier et la valeur du projet lorsque le coût d'investissement, le calendrier de construction et la valeur des résultats sont incertains. Nous illustrons cette approche avec des distributions à queues minces et épaisses des dépassements de coûts et de délais. Enfin, nous montrons comment les praticiens peuvent mettre en œuvre l'approche proposée en utilisant des distributions empiriques des coûts et des calendriers provenant de projets antérieurs comparables.

Mots clés : Options réelles, grands projets, retards, dépassements de coûts, incertitude, prévision par référence à une classe

Acknowledgements: The third author is partially supported by an NSERC Discovery Grant.

1 Introduction

Large capital investment projects, also known as major projects, are crucial to economic and social development (Gil, 2022) but often deviate from their original budget and schedule (Love, Pinto et al., 2022)This is true for transportation infrastructure projects (Love et al., 2019b; Cavalieri et al., 2019),construction projects (Zidane and Andersen, 2018), hydropower mega-projects (Ansar et al., 2014), and IT projects (Flyvbjerg and Budzier, 2011) among other industries. These deviations take the forms of cost overruns, schedule overruns or delays, and benefit shortfalls. Cost overruns and delays are explained, at least in part, by a multitude of technical, economic, psychological, governance, and political causes Chen et al. (2023); Flyvbjerg (2021); Cantarelli et al. (2010); Lind and Brunes (2015); Nguyen et al. (2019); Zhang et al. (2020). Lovallo and Kahneman (2003) suggest adopting an outside view, that is to learn from outcomes of comparable, already concluded, projects to enhance the forecasted outcome of any new venture.

A proposed practical implementation of the outside view is the *Reference Class Forecasting* (RCF) technique Flyvbjerg et al. (2004). RCF has been used for rail, road, tunnel or bridge projects (Baerenbold, 2023; Flyvbjerg, 2006, 2008). As summarised by Ansar et al. (2014), RCF involves three steps: (i) identify a reference class, that is, a group of, already-concluded, projects with similar characteristics to a specific venture, (ii) establish empirical distributions of the parameters of interest (i.e., costs, schedule, ...) for the selected reference class, and (iii) use these empirical distributions to enhance the forecasted outcome of the new venture at hand. By applying RCF to a specific class of major projects, Flyvbjerg and COWI (2004), Flyvbjerg (2006, 2008), Ansar et al. (2014) and Awojobi and Jenkins (2016) estimate an uplift or a down lift to the forecasts of the major project at hand. However, they ignore the flexibility in project timing. They assume that the decision-maker will follow a predetermined project timing, regardless of how events unfold. In this paper, by using the real options theory, we argue that the flexibility in the project timing, when affordable, is a crucial tool that can be used to address the risk of project benefits shortfall while taking into account the cost and schedule uncertainty. Indeed, it is possible to address the uncertainty related to the project benefits by choosing optimally when launching the project based on information from the project output demand. Thus, this paper adds to the traditional applications of RCF in project management, the possibility to optimise the project timing based on historical data on cost and delays. The optimal project timing is determined dynamically by observing the current project output value until it hits a predetermined threshold that depends on the uncertainty surrounding the project cost, schedule, and the project output value. The real options approach shows that postponing the project launch until the project output value hits this specific threshold ensures that the project value is optimised.

The technique used in Love, Sing, Wang et al. (2014), Love, Sing, Carey et al. (2015), Love, Sing, Carey et al. (2015) and Love, Zhou et al. (2017) (*Love's technique* thereafter) consists of fitting theoretical distributions to empirical distributions of cost overruns and delays. It may be seen as an alternative implementation of the outside view approach. Indeed, Love's technique also uses existing information from prior comparable projects. Here again, the fitted theoretical distributions better inform the investment decision. The usual flexibility on the timing of the project remains ignored in Love's technique too. The approach presented in this paper can be implemented using fitted theoretical distributions as long as they have finite moments, as is generally the case for project costs and schedules. However, as it will be further explained, Love's technique is based on data derived from projects' contracts. Therefore projects are already scheduled. On the contrary, RCF relies on data available when the investment decision is being assessed, and projects are not yet definitely scheduled. We find that data derived from the RCF's approach are more coherent with our approach.

We propose an economic model based on the real options theory. Real options theory is useful in recognising the temporal dimension of knowledge Pender (2001). In the 1990s and early 2000s, real options theory received much interest (Borison, 2005) and has been later extensively investigated in several areas, including in infrasturure and transportation projects (Di Maddaloni et al., 2022; Krystallis et al., 2024), mining exploitation (Slade, 2001), energy management (Siddiqui and Fleten, 2010), logging (Ben Abdallah and Lasserre, 2016), research and development (Oriani and Sobrero, 2008), public-private partnership mechanisms (Attarzadeh et al., 2017; Buyukyoran and Gundes, 2018), to mention a few examples.

Our real options model considers probabilistic project cost and schedule as they are sources of anticipated risks. Our model also considers stochastic project output value as a proxy for variations in macro-economic and market risks. The project owner maximises the overall project value by choosing its optimal timing.

The paper is organised as follows. Section 2 introduces the real options theory and describes its main results related to project valuation and project optimal timing. Section 2 also provides a review of findings from the literature related to major projects underperformance in terms of cost and schedule overruns as well as benefits shortfalls. Section 3 introduces a benchmark real options model in which project cost and schedule are deterministic while the project output value follows a geometric Brownian motion. Section 4 discusses the impact of project cost and schedule uncertainty on the project optimal timing and value. Our proposed approach is versatile and can accommodate various theoretical and empirical distributions. We provide illustrations when cost and schedule follow (i) normal distributions, and (ii) upper fat-tailed distributions. Then, we show how our approach can accommodate and complete the RCF approach and be used by decision-makers in practice. We provide a summary of the results and conclude the paper in Section 5.

2 Literature review

Under the traditional financial analyses based on the net present value, a project is deemed profitable when its present value is higher than its investment cost, which is when its net present value is positive. However, starting a project as soon as its net present value becomes positive is risky as the project value can decrease while it is almost impossible to abandon or even suspend its implementation. It is straightforward that, when possible, the investor has to wait until the project's present value becomes sufficiently higher than the project's investment cost to justify an irreversible commitment of resources to the project. The project option value is the sum of the net project value, i.e., the project expected discounted benefits minus the project discounted investment cost, and the value of the project timing flexibility. Using the real options approach, it is possible to determine the project optimal timing by maximising the project option value. Its maximisation balances the gain from making the project service or output available sooner and the possible loss from a sharp decrease in the project value while resources are committed irreversibly. Consequently, a project cannot be deemed viable unless its present value surpasses its cost augmented by the value of the timing flexibility. It has to be noted that the option value increases with the level of uncertainty, usually captured by the variance of the project's present value. The reason is that, while higher uncertainty may imply too high or too low project value, flexibility allows the project owner to launch the project only when higher project value prevails. The option value captures the value of this flexibility. In other terms, the optimal project timing, expressed indirectly as the moment when the project's present value hits a specific threshold, the investment trigger, is further postponed when uncertainty is higher (Dixit, 1992). As summarised in Conejo et al. (2016), when a project owner has the discretion to consider investing at a later date, given the trajectory of the output value, it may be beneficial to delay the investment decision. In doing so, the company trades off the following three aspects in determining the correct timing of the investment: (i) the marginal benefit from postponing the investment cost. Rather than paying the full investment cost now, delaying the project's start implies a lower discounted investment cost. (ii) the marginal benefit from starting the project with a higher output value. It may be profitable to invest immediately, but the output value's trajectory may be such that it is beneficial to delay investment. (iii) the marginal cost from foregone benefits during the waiting period. The benefits that the project owner could have been earning are an opportunity cost that must be figured into its decision.

Based on the real options approach, many scholars provided theoretical frameworks for the valuation and the optimal timing of investments under uncertainty (Dixit and Pindyck, 1994). In their pioneer paper, McDonald and Siegel (1986) consider an output price and a cost of investing that follow stochastic processes, specifically geometric Brownian motions. They show that waiting can be of significant value and that for reasonable parameter values, the investment decision must be postponed until benefits are almost twice higher than the investment cost. This is in sharp contradiction to the traditional financial valuation under which the project is justified as soon as the benefits equal the investment cost. In these initial real options models, it is assumed that the project construction is instantaneous, meaning that the project benefits are immediately available once the decision to invest is made. Progressively, time-to-build, known in project management as the project duration or schedule, was introduced under different assumptions. Assuming that construction can be halted and later resumed without cost, Majd and Pindyck (1987), confirmed by Milne and Whalley (2000), find that a more extended project schedule magnifies the depressive effect of uncertainty on investment. More precisely, higher uncertainty related to project value further postpones project launching for longer time-to-build. Expecting a longer project schedule is an incentive to delay the project execution. In this paper, we investigate further the impact of the uncertainty related to the project value, cost, and schedule on both project value and timing.

As mentioned earlier, the deviation of major projects from initial plans in terms of budgeted cost and project duration or time-to-build has been extensively studied in the project management literature. Major projects often witness considerable underperformance that Flyvbjerg (2014) coined the “iron law of megaprojects. Ansar et al. (2014) show that if large dams planners are willing to accept a maximum of 20% risk of a cost overrun, they should accept almost double the expected construction cost and a maximum of 50% chance of a cost overrun requires 26% cost uplift. To cost overruns and delays, Flyvbjerg (2007) adds errors in forecasting demands such as in rail projects where actual traffic is on average 40% lower than forecast traffic. In road projects, actual traffic is on average 10% lower than forecast. Pinheiro Catalão et al. (2019) found that central governments incur on an average cost overrun of 23% and local governments on 6%. As demonstrated by Kahneman and Tversky (1977) , human judgment is generally optimistic due to overconfidence and insufficient consideration of available data on similar projects. This behaviour, described as the *planning fallacy* Lovallo and Kahneman (2003) is said to be due to the “inside view” of the project stakeholders. On the contrary, an “outside view” consists of considering systematically similar projects when it is time to evaluate project cost and time-to-build, and to forecast the demand for their products and services.

Costs deviation from the budget can take the forms of cost over- and underruns. Cost overruns can be found in various economic sectors, including construction, hydropower, IT and transportation projects. Explanations of cost overruns in the transportation sector include scope changes, rework and price escalation (e.g., Terrill (2016); Locatelli et al. (2017); Love et al. (2017); Welde and Odeck (2017)) and the previously discussed planning fallacy (Flyvbjerg et al., 2002, 2003, 2004, 2009; Flyvbjerg, 2013) . Deviations from budgeted costs can also be in the form of cost underruns. Love et al. (2019a) al. (2019) find that cost underruns are almost equally prevalent in their transportation project dataset. Chapman (2024) evaluated the project delivery performance of National Highways, the government company responsible for delivering England’s road investment strategy which completed 138 road/highway major schemes over 16 years between 2001 and 2016. He found that 2.4% of project suffered cost overrun with 42% of projects delivered on or under budgeted cost.The consideration of cost uncertainties suggested in this paper is equally relevant whether cost overruns prevail or are as likely as underruns. Over- and underruns translate into different assumptions on the mean of the distribution of cost deviations.

The debate as to whether cost overruns prevail is rooted in part on how cost deviations are defined and measured. Flyvbjerg et al. (2018, p.175) offer the following definition :

> Cost overrun is the amount by which actual cost exceeds estimated cost, with cost measured in the local currency, constant prices, and against a consistent baseline. Overrun

> is typically measured in per cent of estimated cost, with a positive value indicating cost overrun and a negative value underrun. Size, frequency, and distribution of cost overrun should all be measured as part of measuring cost overrun for a certain investment type. Cost overrun is the difference between actual and estimated capital costs for an investment. The difference may be measured in absolute or relative terms. In absolute terms, cost overrun is measured as actual minus estimated cost. In relative terms, overrun is measured as either (a) actual cost in per cent of estimated cost, or (b) the ratio of actual divided by estimated cost.

The estimated cost is generally assessed at two different points in time during the investment cycle, that is either when the investment decision is made or at contracting. The cost estimation based on contracts is more accurate than the initial cost estimation, but the latter is more relevant as it justifies the investment (Flyvbjerg et al., 2018). The initial cost and time-to-build estimates are considered for instance in Flyvbjerg et al. (2002, 2004), Flyvbjerg et al. (2009), Cantarelli et al. (2010), Flyvbjerg and Budzier (2011), Flyvbjerg (2014) and Ansar et al. (2014). Love and Ahiaga-Dagbui (2018) object to this approach, they claim (p. 363) that, "The use of the budget at the decision-to-build may lead to inflated cost overruns being propagated". They recommend using the budget at contracting as a baseline instead, which would, on average, show a lower cost overrun because this baseline is placed later in the investment cycle. As the proposed real options model is intended to consider the flexibility offered to the decision-makers for the project timing, we adopt thereafter the definition proposed by Flyvbjerg et al. (2018).

There is a consensus over the utility of reinforcing the implementation of the outside view in project management. The traditional approach, Reference Class Forecasting (RCF), consists of adjusting project cost, schedule, and project output demand forecast to their most likely values based on similar completed projects. However, RCF remains a static approach because its outcome is not conditional to the information made available after the date of the decision to invest. In that, it fails in considering the flexibility in the timing of the investment and under-estimates the project value especially when the project value is volatile. The real options theory is a dynamic decision-making approach that optimises project value and timing, taking into account future information based on the stochastic process followed by the project value. In situations of high uncertainty, the dynamic approach is much more accurate. In this paper, we consider the uncertainty surrounding project cost and schedule, represented by distribution parameters that do not vary with time. Costs and schedule deviations distribution are generally found to be fat upper tailed. Hence, Flyvbjerg et al. (2018, p. 181) write "Studies of cost overrun in large capital investment projects show that average overrun is typically higher than median overrun, indicating fat upper tails". We show how to factor in both thin and fat upper tailed distributions of costs and schedule deviations.

3 A benchmark model with deterministic cost and schedule

In this section, we introduce a real options model with time-to-build and deterministic cost and schedule. Consider a project owner who wants to optimise the value and the timing of an investment project. We assume that once completed after some period defined as the project schedule D, the project will produce one unit of output per unit of time during its operational period T. We assume that once the project construction has started, it cannot be abandoned or suspended and that the project investment cost I has to be entirely incurred at the beginning of the construction period.[1,2] We assume that the project output value p_t, net of any operational costs follows a Brownian geometric motion with drift

[1]Even though valuable real options may be available during the project schedule, such as the option to abandon or to suspend the project, we assume that, in practice, the main goal of a major project owner once construction has started is to complete the project.

[2]If the investment costs are paid by instalment, as is common in major projects, the project investment cost I is simply the sum of the instalment payments discounted when the construction starts.

μ and volatility σ:

$$dp_t = \mu p_t dt + \sigma p_t dz \tag{1}$$

where $dz = \varepsilon\sqrt{dt}$ is the increment of Wiener process, and ε is the standardised Gaussian white noise. We assume that $\delta = r - \mu > 0$, where r is the discount rate. Let p denote the output price at time 0 ($p = p_0$).

The project option value is the maximum of the project's present value obtained by deciding the optimal project timing τ^* to start construction, that is

$$\text{V(p)=Ma}x_{\tau=\tau^*} E_0 \left[\int_{\tau+D}^{\tau+D+T} e^{-rt} p_t dt - e^{-r\tau} I \right] \tag{2}$$

where E_0 represents the investor expectation at time 0.

$V(p)$ can be derived as in Dixit & Pindyck (1994). It satisfies the following Bellmann equation in the waiting region,

$$V(p) = e^{-rdt} E_{dt} V(p + dp).$$

Using Itô's lemma, one can show that $V(p)$ is the solution to the partial differential equation

$$\frac{\sigma^2}{2} p^2 V_{pp} + \mu p V_p - rV = 0$$

with $lim_{p\to 0} V(p) = 0$. Therefore, $V(p)$ is of the form $V(p) = Ap^\beta$, where A is a positive constant and $\beta > 1$ is the positive solution of the fundamental quadratic equation $\frac{\sigma^2}{2}x^2 + (\mu - \frac{\sigma^2}{2})x - r = 0$. Besides, $V(p)$ satisfies the following Value-Matching and Smooth-Pasting conditions at τ^* when the project's present value is p^*:

- Value-matching condition, when the decision to invest is made, is

$$Ap^{*\beta} = E_0 \left[\int_D^{D+T} e^{-rt} p_t dt \right] - I \tag{3}$$

  Given that $E_0\left(p_t\right) = p^* e^{\mu t}$, then the Value-Matching condition is

$$Ap^{*\beta} = \frac{1 - e^{-\delta T}}{\delta} e^{-\delta D} p^* - I \tag{4}$$

- At the same moment, the Smooth-Pasting condition is

$$\beta A p^{*\beta - 1} = \frac{1 - e^{-\delta T}}{\delta} e^{-\delta D} \tag{5}$$

It is now possible to determine the threshold price p^* triggering the project (the trigger price or investment trigger) and the constant A. We obtain $A = \frac{I}{\beta-1} p^{*-\beta}$, and

$$V(p) = \frac{I}{\beta - 1} \left(\frac{p}{p^*} \right)^\beta \text{ if } p \le p^* \tag{6}$$

$$V(p) = \frac{1 - e^{-\delta T}}{\delta} e^{-\delta D} p - I \text{ if } p > p^* \tag{7}$$

$$p^* = \frac{\delta}{1 - e^{-\delta T}} \frac{\beta}{\beta - 1} I e^{\delta D} \tag{8}$$

Thus, in the waiting region defined by $p \leq p^*$, the project option, whose value is given by Equation (6), must be maintained alive until the output value hits the trigger value p^*. Then, the option must be exercised, giving birth to the project whose value is given by Equation (7).

Equation (8) shows that an increase in the project cost I or schedule D, implies a higher trigger price to justify committing resources to the project. Therefore, expecting an increase in the project cost or schedule before initiating a project can justify its delay to wait for higher expected project outcomes. The next section deals more formally with the impact on the project value and optimal timing of project cost and schedule uncertainty.

This optimization problem with a schedule D happens to reach the same conclusions as to the problem without time-to-build (instantaneous construction), provided that the threshold value is multiplied by $e^{-\delta D}$. The factor $e^{-\delta D}$ is the product of the discounting factor e^{-rD} and of $e^{\mu D}$. The latter factor captures the increase in the project price during the construction period D. This observation, proved in Annex A.1, will be used in the next section.

4 Optimal timing and capacity when time-to-build and investment cost are uncertain

4.1 General case

In this section, we assume more realistically that the project schedule, now called d, and cost, now called i, are uncertain. We denote by $\mathbb{E}$ the expectation operator with respect to the distribution laws of d and i before the decision to invest is made. The expectation operator $\mathbb{E}$ has to be distinguished from E_0, the expectation operator with respect to the process p_t. At this stage, no specific assumptions are made on the distribution laws. They are assumed known and remain unchanged as long as the project execution has not started. Once the construction begins, more information is obtained concerning the project schedule and cost. Still, this information has no impact on the project as the decision to invest is assumed irreversible. Finally, we assume that the project uncertain cost and schedule are independent of the process $(p_t : t \geq 0)$. The project option value is therefore:

$$V_d(p) = \sup_{\tau} E_0\, \mathbb{E} \left[\int_{\tau+d}^{\tau+d+T} e^{-rt} p_t dt - i e^{-r\tau} \right] \tag{9}$$

Theorem 4.1. gives a formula for the expression of $V_d(p)$. Its proof is given in Appendix A.2.

Theorem 4.1. *Suppose that the distributions of d and i are such that $\mathbb{E}[i]$ and $\mathbb{E}\left[e^{-\delta d}\right]$ are both finite. Then*

$$V_d(p) = \sup_{\tau} E_0 \left[\int_{\tau}^{\tau+T} e^{-rt} \left(\mathbb{E}e^{-\delta d}\right) p_t dt - \mathbb{E}\left(i\right) e^{-r\tau} \right] \tag{10}$$

Theorem 4.1 shows that the optimal timing and valuation of a project, whose schedule d and cost i are uncertain, can be transformed into the optimal timing and valuation of an instantaneous project provided that the project output value is multiplied by $\mathbb{E}e^{-\delta d}$ and the project cost is replaced by its expected value $\mathbb{E}(i)$. Therefore, it is possible to determine $V_d(p)$ and p_d^* by analogy with the optimisation problem's solution when the time-to-build and the investment cost are deterministic. We obtain

$$V_d(p) = \frac{\mathbb{E}\left(i\right)}{\beta - 1} \left(\frac{p}{p_d^*}\right)^{\beta} \text{ if } p \leq p_d^* \tag{11}$$

$$V_d(p) = \frac{1 - e^{-\delta T}}{\delta} p\mathbb{E}\left(e^{-\delta d}\right) - \mathbb{E}\left(i\right) \text{ if } p > p_d^* \tag{12}$$

$$p_d^* = \left(1 - e^{-\delta T}\right) \frac{\beta}{\beta - 1} \frac{\mathbb{E}\left(i\right)}{\mathbb{E}\left(e^{-\delta d}\right)} \tag{13}$$

Let us write $m_I = \mathbb{E}\left[i\right]$ and define the function $\phi \colon \mathbb{R} \longrightarrow [0, \infty]$ by $\phi(u) = \mathbb{E}\left[e^{ud}\right]$. Then we have

$$V_d(p) = \begin{cases} \frac{m_I}{\beta-1}\left(\frac{p}{p_d^*}\right)^{\beta} & \text{if} \quad p \leq p_d^*, \\ \frac{1-e^{-\delta T}}{\delta} p\phi\left(-\delta\right) - m_I & \text{if} \quad p > p_d^*, \end{cases} \tag{14}$$

where

$$p_d^* = \left(1 - e^{-\delta T}\right) \frac{\beta}{\beta - 1} \frac{m_I}{\phi(-\delta)} \tag{15}$$

Equation (14) provides the expressions of the optimal option value (when $p \leq p_d^*$) and the optimal project value (when $p > p_d^*$) in the realistic situation for a decision-maker where both the project cost and schedule are uncertain while its output value is stochastic. Equation (15) provides the expression of the trigger price that determines the optimal timing of the investment. The following section will illustrate these results when the project cost and schedule follow specific distribution laws.

4.2 Assuming a thin-tailed distribution: Gaussian case

We first evaluate (14) and (15) when $\left(\tilde{i}, \tilde{d}\right)$ follow a joint normal distribution with mean (m_I, m_D) and a covariance matrix

$$\Sigma = \begin{pmatrix} \sigma_I^2 & \rho\sigma_I\sigma_D \\ \rho\sigma_I\sigma_D & \sigma_D^2 \end{pmatrix},$$

For any value of ρ, we have

$$\phi\left(u\right) = \exp\left\{m_D u + \frac{1}{2}\sigma_D^2 u^2\right\}. \tag{16}$$

Substituting (16) back into (14) and (15) , we obtain

$$V_d(p) = \begin{cases} \frac{m_I}{\beta-1}\left(\frac{p}{p_d^*}\right)^{\beta} & \text{if} \quad p \leq p_d^*, \\ \frac{1-e^{-\delta T}}{\delta} p e^{\sigma_D^2 \delta^2/2 - \delta m_D} - m_I & \text{if} \quad p > p_d^*, \end{cases} \tag{17}$$

and

$$p_d^* = \left(1 - e^{-\delta T}\right) \frac{\beta}{\beta - 1} m_I e^{\delta m_D - \sigma_D^2 \delta^2/2} \tag{18}$$

Equation (17) expresses the optimal value of the option (when $p \leq p_d^*$) and of the project (when $p > p_d^*$). In Table 1, we highlight the impact of an increase in the parameters of the normal distributions of cost and schedule on the project timing, through the trigger price p^*, and on the project value $V_d(p)$.[3]

Table 1: Impact of an increase in the parameters of the normal distributions

Parameters	Impact on project timing	Impact on project value
m_D	Positive	Negative
σ_D	Negative	Positive
m_I	Positive	Negative

[3]Decreases in the parameters have a symmetric impact.

When the expected project schedule m_D increases, meaning that the project execution and benefits are expectedly delayed, the project value decreases due to a longer discounting period. The decrease in the project value translates into an increase in the trigger price, meaning that the decision to invest has to be delayed to allow additional growth in the project output value before committing any resources to its development.

The standard deviation of the schedule σ_d affects the trigger price, option, and project values because expressing the expected value of the discount factor over the period d, $\mathbb{E}\left(e^{-\delta d}\right)$, requires the use of the moment generating function of the distribution of d, $\phi(u)$, that depends on σ_d. In other words, contrary to the cost overrun, it is not only the expected schedule that enters into the expression of the trigger price, the option and project values but the expectation of a function of d. We have $\mathbb{E}\left(e^{-\delta d}\right) = \phi(-\delta) = \exp\left\{-\delta m_D + \frac{1}{2}\sigma_D^2\delta^2\right\}$ that increases with σ_d [4] increasing the discounted revenues. As the present value of the investment cost is not affected, the project value increases. Besides, as $\mathbb{E}\left(e^{-\delta d}\right)$ increases, the trigger price decreases, leading to an increase in the stochastic discount factor $\left(\frac{p}{p_d^*}\right)^\beta$ and, as a consequence, an increase in the option value.

An increase in the expected cost overrun has a negative impact on the project value. It has a positive effect on the trigger price. Indeed, a higher trigger price delays the project to offset the additional cost optimally.

Note that the expected value and the schedule's volatility have an opposite impact on the timing and the project value. In the next section, we illustrate our approach with upper fat-tailed distributions.

4.3 Assuming a fat-tailed distribution

As mentioned earlier, the distributions of costs and schedule deviations are generally found to have fat upper tails (Flyvbjerg et al. 2018). As explained by Cooke et al. (2011), mathematicians have used at least three main definitions of fat-tailed distributions. Some texts refer "leptokurtic distributions", that is, distributions whose extreme values are "more probable than normal", as fat-tailed distributions. These are distributions with excess kurtosis greater than zero, and whose tails go to zero slower than the normal distribution. This definition is in line with the frequent understanding of the meaning of fat-tailed distributions by economists. We consider exponential distributions as fat-tailed distributions as their tails asymptotically approach zero more slowly than a normal distribution.[5] Exponential distributions produce more outliers than the normal distribution. We analyse (14) when i and d are exponentially distributed random variables, with parameters λ_I and λ_D, respectively. Under these hypotheses, the expected cost overrun is $m_I = \frac{1}{\lambda_I}$ with standard deviation $\sigma_I = \frac{1}{\lambda_I}$, and the expected delay is $m_D = \frac{1}{\lambda_D}$ with standard deviation $\sigma_D = \frac{1}{\lambda_D}$. For $u < \lambda_D$,

$$\phi(u)=\frac{\lambda_D}{\lambda_D - u}\ . \tag{19}$$

In this case, (14) and (15) yield:

$$V_d(p) = \begin{cases} \frac{1}{\lambda_I(\beta-1)}\left(\frac{p}{p_d^*}\right)^\beta & \text{if} \quad p \le p_d^*, \\ \frac{1-e^{-\delta T}}{\delta} p \frac{\lambda_D}{\lambda_D+\delta} - \frac{1}{\lambda_I} & \text{if} \quad p > p_d^*, \end{cases} \tag{20}$$

where

$$p_d^* = \left(1 - e^{-\delta T}\right)\frac{\beta}{\beta-1}\frac{\lambda_D + \delta}{\lambda_D\lambda_I}. \tag{21}$$

In Table 2, we highlight the impact of an increase in the cost and schedule exponential distributions parameters on the project timing, through the trigger price p^*, and on the project value $V_d(p)$[6].

[4] $\frac{\partial\left(\exp\left\{-\delta m_D + \frac{1}{2}\sigma_D^2\delta^2\right\}\right)}{\partial\sigma_D} = \sigma_D\delta^2\exp\left\{-\delta m_D + \frac{1}{2}\sigma_D^2\delta^2\right\} = \sigma_D\delta^2\mathbb{E}\left(e^{-\delta d}\right) > 0$

[5] Exponential distributions have an excess kurtosis of 6.

[6] Decreases in the parameters have a symmetric impact.

Table 2: Impact of an increase in the parameters of the exponential distributions

Parameters	Impact on project timing	Impact on project value
$m_D, \sigma_D(1/\lambda_D)$	Positive	Negative
$m_I(1/\lambda_I)$	Positive	Negative

Similarly to the normal distribution laws, the impact of higher expected investment cost on the project's timing is positive, and the impact on the project value is negative. The same explanation applies. However, the effect of an increase in the expected schedule value and an increase in its variability is negative on the project value. In this situation, the impact of an increase in the expected schedule more than offsets the opposite effect of an identical rise in the schedule's variability.

4.4 Reference class forecasting with optimal timing

Our proposed framework can be interestingly combined with the Reference Class Forecasting (RCF) approach. In RCF, empirical distributions for cost and schedule overruns are computed by looking at similar completed projects. Projects are deemed comparable when they are similar in scope and when risk of cost and schedule overruns can be treated as statistically identical. These comparable and completed projects are then said to be of the same "reference class" as the project at hand (Flyvbjerg and COWI 2004). For completed projects, based on historical records, cost overrun is computed as the deviation of the actual cost from the estimated cost.[7] Schedule overrun is computed as the deviation of the actual schedule from the estimated schedule. Cumulative distributions for both cost and schedule overruns for projects in the same reference class can then be established. It is beyond the scope of this paper to present the RCF approach in detail. Flyvbjerg and COWI (2004) offer a detailed description of the implementation of the method for transportation projects in the UK. Awojobi and Jenkins (2016) show how to apply RCF for hydropower projects from the World Bank.[8] Ansar et al. (2014) show how to determine empirical distributions for cost and schedule overrun of hydropower megaprojects. Sovacool et al. (2014) determine empirical distributions of cost overruns for hydroelectric dams, nuclear power plants, wind farms, and solar facilities.

In the framework proposed in this paper, the project schedule can be expressed as $d = D + \tilde{d}$ with D the deterministic estimated schedule and $\tilde{d}$ the uncertain delay. Investment cost can be expressed as $i = I + \tilde{i}$ with I the deterministic estimated project cost and $\tilde{i}$ the uncertain cost overrun. The RCF approach allows to determine the empirical moments of $\tilde{d}$ and $\tilde{i}$ from which we can deduct the empirical moments of d and i.

Using the empirical distributions obtained from the RCF approach, we can estimate the parameters m_I and $\phi(-\delta)$. To see how this works, consider a random variable X and suppose that N samples have been drawn according to X's law: $x_1, ..., x_N$. These samples can be used to estimate the expectation of any function f of X. In order to estimate $\mathbb{E}[f(X)]$, calculate

$$\widehat{f} = \frac{1}{N} \sum_{j=1}^{n} f(x_j). \tag{22}$$

Suppose an empirical distribution for the delay and cost overrun has been identified according to the RCF framework from N similar projects. We obtain N cost overruns $i_1, ..., i_N$ and N delays $d_1, ..., d_N$. Using these values, we can estimate the parameter m_I by setting $X = d$ and $f(x) = x$ in

[7] All costs are computed in real terms, that is, corrected for inflation. This deviation can be computed as a percentage of the estimated cost.

[8] They compute the overall net cost of time overrun rather than schedule overrun per se. In their study, the cost of time overrun is distinguished from cost overrun.

equation (22) to obtain

$$\widehat{m}_I = I + \frac{1}{N}\sum_{j=1}^{N} i_j \tag{23}$$

Similarly we can estimate $\phi(-\delta)$ by choosing $f(x) = e^{-\delta x}$ and $X = i$ in equation (22), which yields

$$\widehat{\phi} = \frac{1}{N}\sum_{j=1}^{N} e^{-\delta(D+d_j)} = \frac{1}{N}e^{-\delta D}\sum_{j=1}^{N} e^{-\delta d_j} \tag{24}$$

We then obtain the following formulas for V_d and p^* :

$$V_d(p) = \begin{cases} \frac{\widehat{m}_I}{\beta-1}\left(\frac{p}{p_d^*}\right)^{\beta} & \text{if} \quad p \le p_d^*, \\ \frac{1-e^{-\delta T}}{\delta} p\widehat{\phi} - \widehat{m}_I & \text{if} \quad p > p_d^*, \end{cases} \tag{25}$$

where

$$p_d^* = \left(1 - e^{-\delta T}\right)\frac{\beta}{\beta - 1}\frac{\widehat{m}_I}{\widehat{\phi}} \tag{26}$$

Hence, using the empirical distributions of delays and cost overruns obtained from the RCF approach, we have shown how, using the real options approach, practitioners can optimise the project investment timing and its value that takes into account the timing flexibility. In fact, practitioners must optimally invest as soon as the output price reaches the trigger price. At that time, the project value is given by equation (39) when $p > p_d^*$.

5 Conclusion

The variability of major project costs and schedules are sources of anticipated risks that have been extensively studied. In addition to the uncertainty surrounding project cost and schedule, this paper explicitly considers the uncertainty surrounding the project output value and uses the real options approach to determine the optimal project timing. Precisely, we propose a real options model that encompasses both probabilistic project cost and schedule as well as stochastic project output value. The model allows the analysis of the impact of these three sources of uncertainty (i.e., project cost, schedule, and output value) on the overall project value and optimal timing. We show how different probability distributions (thin- and fat-tailed) affect optimal timing, option, and project values. We perform comparative static analysis on the moments of the distributions of schedule and cost overruns to show the impact of changes on optimal timing, option and project values. We conclude by showing how our proposed approach can be implemented in practice to complement the RCF approach. In doing so, practitioners can use information from similar completed projects to optimise investment timing, option, and project value in situations where both project cost, schedule and value are uncertain.

A Appendices

A.1 Proof of the similarity between the problems with and without time-to-build

Modifying the bounds of the integral in equation (2), we have

$$V(p) = Max_{\tau=\tau^*} E_0\left(\int_{\tau+D}^{\tau+T+D} e^{-rt} p_t dt - e^{-r\tau} I\right) \tag{27}$$

Taking the expectation at time τ of the output value at time $t+D$, we find

$$V(p) = Max_{\tau=\tau^*} E_0 \left(\int_{\tau}^{\tau+T} e^{-r(t+D)} E_\tau p_{t+D} dt - e^{-r\tau} I \right) \tag{28}$$

Using $E_\tau p_{t+D} = p_\tau e^{\mu(t+D-\tau)}$ and substituting, we have

$$V(p) = Max_{\tau=\tau^*} \left[E_0 \left(\int_{\tau}^{\tau+T} e^{-r(t+D)} p_\tau e^{\mu(t+D-\tau)} dt \right) - E_0 \left(e^{-r\tau} I \right) \right] \tag{29}$$

Factoring by $e^{-\delta D}$, we find

$$V(p) = Max_{\tau=\tau^*} \left[e^{-\delta D} E_0 \left(\int_{\tau}^{\tau+T} e^{-rt} p_\tau e^{\mu(t-\tau)} dt \right) - E_0 \left(e^{-r\tau} I \right) \right] \tag{30}$$

Using $p_\tau e^{\mu(t-\tau)} = E_\tau p_t$ and substituting leads to

$$V(p) = Max_{\tau=\tau^*} \left[e^{-\delta D} E_0 \left(\int_{\tau}^{\tau+T} e^{-rt} E_\tau p_t dt \right) - E_0 \left(e^{-r\tau} I \right) \right] \tag{31}$$

which is

$$V(p) = Max_{\tau=\tau^*} \left[e^{-\delta D} E_0 E_\tau \left(\int_{\tau}^{\tau+T} e^{-rt} p_t dt \right) - E_0 \left(e^{-r\tau} I \right) \right] \tag{32}$$

As $\tau > 0$, we have

$$V(p) = Max_{\tau=\tau^*} E_0 \left[\int_{\tau}^{\tau+T} e^{-rt} \left(e^{-\delta D} p_t \right) dt - e^{-r\tau} I \right] \tag{33}$$

A.2 Proof of Theorem 4.1

We have

$$E_0, \mathbb{E} \left[\int_{\tau+d}^{\tau+d+T} e^{-rt} p_t dt - i e^{-r\tau} \right] = E_0 \left[\mathbb{E} \left(\int_{\tau+d}^{\tau+d+T} e^{-rt} p_t dt \right) - \mathbb{E}(i) e^{-r\tau} \right] \tag{34}$$

For ease of notation, we call $A = E_0 \left[\mathbb{E} \left(\int_{\tau+d}^{\tau+d+T} e^{-rt} p_t dt \right) \right]$ the first term of the right-hand side of (34). By switching the expectation operators and then changing the bounds of the integral, we find

$$A = \mathbb{E} \left[E_0 \left(\int_{\tau+d}^{\tau+d+T} e^{-rt} p_t dt \right) \right] \tag{35}$$

$$= \mathbb{E} \left[E_0 \left(\int_{\tau}^{\tau+T} e^{-r(t+d)} p_{t+d} dt \right) \right], \tag{36}$$

Then, switching the expectation operators and appealing to the martingale property of Brownian motion, we have

$$A = E_0 \left[\mathbb{E} \left(\int_{\tau}^{\tau+T} e^{-r(t+d)} p_{t+d} dt \right) \right] \tag{37}$$

$$= E_0 \left(E_\tau \left[\mathbb{E} \left(\int_\tau^{\tau+T} e^{-r(t+d)} p_{t+d} dt \right) \right] \right) \tag{38}$$

By switching the last two expectation operators, we find

$$A = E_0 \left(\mathbb{E} \left[E_\tau \left(\int_\tau^{\tau+T} e^{-r(t+d)} p_{t+d} dt \right) \right] \right) \tag{39}$$

$$= E_0 \left[\mathbb{E} \left(\int_\tau^{\tau+T} e^{-r(t+d)} E_\tau p_{t+d} dt \right) \right] \tag{40}$$

$$= E_0 \left[\mathbb{E} \left(\int_\tau^{\tau+T} e^{-r(t+d)} p_\tau e^{\mu(t+d-\tau)} dt \right) \right] \tag{41}$$

$$= E_0 \left[\mathbb{E} \left(\int_\tau^{\tau+T} e^{-r(t+d)} e^{\mu d} \left[p_\tau e^{\mu(t-\tau)} \right] dt \right) \right] \tag{42}$$

Since $E_\tau p_t = p_\tau e^{\mu(t-\tau)}$ for $t \geq \tau$, we obtain

$$A = \left[\mathbb{E} e^{-\delta d} \right] E_0 \left(\int_\tau^{\tau+T} e^{-rt} \left[E_\tau p_t \right] dt \right) \tag{43}$$

$$= E_0 \left(\int_\tau^{\tau+T} e^{-rt} \left[\mathbb{E} e^{-\delta d} \right] p_t dt \right) \tag{44}$$

Therefore,

$$E_0 \mathbb{E} \left[\int_{\tau+d}^{\tau+d+T} e^{-rt} p_t dt - i e^{-r\tau} \right] = E_0 \left(\int_\tau^{\tau+T} e^{-rt} \left[\mathbb{E} e^{-\delta d} \right] p_t dt - \mathbb{E}(i) e^{-r\tau} \right) \tag{45}$$

$$= E_0 \left[\int_\tau^{\tau+T} e^{-rt} \left[\mathbb{E} e^{-\delta d} \right] p_t dt - \mathbb{E}(i) e^{-r\tau} \right]. \tag{46}$$

References

Ansar, A., Flyvbjerg, B., Budzier, A., and Lunn, D. (2014). Should we build more large dams? The actual costs of hydropower megaproject development. Energy Policy, 69:43–56.

Attarzadeh, M., Chua, D. K., Beer, M., and Abbott, E. L. (2017). Options-based negotiation management of PPP–BOT infrastructure projects. Construction Management and Economics, 35(11-12):676–692.

Awojobi, O. and Jenkins, G. P. (2016). Managing the cost overrun risks of hydroelectric dams: An application of reference class forecasting techniques. Renewable and Sustainable Energy Reviews, 63:19–32.

Baerenbold, R. (2023). Reducing risks in megaprojects: The potential of reference class forecasting. Project Leadership and Society, page 100103.

Ben Abdallah, S. and Lasserre, P. (2016). Asset retirement with infinitely repeated alternative replacements: Harvest age and species choice in forestry. Journal of Economic Dynamics and Control, 70:144–164.

Borison, A. (2005). Real Options Analysis: Where are the Emperor's Clothes? Journal of Applied Corporate Finance, 17(2):17–31.

Buyukyoran, F. and Gundes, S. (2018). Optimized real options-based approach for government guarantees in PPP toll road projects. Construction Management and Economics, 36(4):203–216.

Cantarelli, C. C., Flyvbjerg, B., Molin, E. J., and van Wee, B. (2010). Cost overruns in large-scale transportation infrastructure projects: Explanations and their theoretical embeddedness. European Journal of Transport and Infrastructure Research, 10(1):5–18.

Cavalieri, M., Cristaudo, R., and Guccio, C. (2019). Tales on the dark side of the transport infrastructure provision: a systematic literature review of the determinants of cost overruns. Transport Reviews, 39(6):774–794.

Chapman, P. (2024). Project delivery performance: Insights from english roads major schemes. Project Leadership and Society, page 100128.

Chen, Y., Ahiaga-Dagbui, D. D., Thaheem, M. J., and Shrestha, A. (2023). Toward a deeper understanding of optimism bias and transport project cost overrun. Project Management Journal, 54(5):561–578.

Conejo, A. J., Baringo, L., Jalal Kazempour, S., and Siddiqui, A. S. (2016). Investment in electricity generation and transmission: Decision making under uncertainty. Switzerland: Springer International Publishing, Cham Zug, Switzerland, springer i edition.

Di Maddaloni, F., Favato, G., and Vecchiato, R. (2022). Whether and when to invest in transportation projects: Combining scenarios and real options to manage the uncertainty of costs and benefits. IEEE Transactions on Engineering Management.

Dixit, A. (1992). Investment and hysteresis. Journal of Economic Perspectives, 6(1):107–132.

Dixit, A. K. and Pindyck, R. S. (1994). Investment under uncertainty. Princeton University Press, Princeton, NJ.

Flyvbjerg, B. (2006). From Nobel Prize to Project Management: Getting Risks Right. Project Management Journal, 37(5):5–16.

Flyvbjerg, B. (2007). Cost overruns and demand shortfalls in urban rail and other infrastructure. Transportation Planning and Technology, 30(1):9–30.

Flyvbjerg, B. (2008). Curbing optimism bias and strategic misrepresentation in planning: Reference class forecasting in practice. European Planning Studies, 16(1):3–21.

Flyvbjerg, B. (2013). Quality control and due diligence in project management: Getting decisions right by taking the outside view. International Journal of Project Management, 31(5):760–774.

Flyvbjerg, B. (2014). What you should know about megaprojects and why: An overview. Project Management Journal, 45(2):6–19.

Flyvbjerg, B. (2021). Top ten behavioral biases in project management: An overview. Project Management Journal, 52(6):531–546.

Flyvbjerg, B., Ansar, A., Budzier, A., Buhl, S., Cantarelli, C., Garbuio, M., Glenting, C., Holm, M. S., Lovallo, D., Lunn, D., Molin, E., Rønnest, A., Stewart, A., and van Wee, B. (2018). Five things you should know about cost overrun. Transportation Research Part A: Policy and Practice, 118(July):174–190.

Flyvbjerg, B. and Budzier, A. (2011). Why your IT project may be riskier than you think. Harvard Business Review, 89(9):23–25.

Flyvbjerg, B. and COWI (2004). Procedures for dealing with optimism bias in transport planning: Guidance document 2006-09. The British Department for Transport, (June):61.

Flyvbjerg, B., Garbuio, M., and Lovallo, D. (2009). Delusion and deception in large infrastructure projects: two models for explaining and preventing executive disaster. California Management Review, 51(2):170–194.

Flyvbjerg, B., Holm, M. K., and Buhl, S. L. (2003). How common and how large are cost overruns in transport infrastructure projects? Transport Reviews, 23(1):71–88.

Flyvbjerg, B., Holm, M. S., and Buhl, S. (2002). Underestimating costs in public works projects: Error or lie? Journal of the American Planning Association, 68(3):279–295.

Flyvbjerg, B., Holm, M. S., and Buhl, S. L. (2004). What Causes Cost Overrun in Transport Infrastructure Projects? Transport Reviews, 24(1):3–18.

Gil, N. (2022). Megaprojects – a meandering journey towards a theory of purpose, value creation and value distribution. Construction Management and Economics, 40(7-8):562–584.

Kahneman, D. and Tversky, A. (1977). Intuitive prediction: Biases and corrective procedures. Technical report.

Krystallis, I., Laraqui Mahi, Z. A. A., and Di Maddaloni, F. (2024). Flexible infrastructure design: A real options reasoning approach to navigating uncertainty in large-scale projects. Journal of Management in Engineering, 40(3):04024012.

Lind, H. and Brunes, F. (2015). Explaining cost overruns in infrastructure projects: A new framework with applications to Sweden. Construction Management and Economics, 33(7):554–568.

Locatelli, G., Invernizzi, D. C., and Brookes, N. J. (2017). Project characteristics and performance in Europe: An empirical analysis for large transport infrastructure projects. Transportation Research Part A: Policy and Practice, 98:108–122.

Lovallo, D. and Kahneman, D. (2003). Delusions of success. Harvard Business Review, 81(7):56–63.

Love, P. E., Ika, L. A., and Ahiaga-Dagbui, D. D. (2019a). On de-bunking 'fake news' in a post truth era: Why does the Planning Fallacy explanation for cost overruns fall short? Transportation Research Part A: Policy and Practice, 126:397–408.

Love, P. E., Pinto, J. K., and Ika, L. A. (2022). Hundreds of years of pain, with minimal gain: capital project cost overruns, the past, present, and optimistic future. IEEE engineering management review, 50(4):56–70.

Love, P. E., Sing, C. P., Carey, B., and Kim, J. T. (2015). Estimating construction contingency: Accommodating the potential for cost overruns in road construction projects. Journal of Infrastructure Systems, 21(2):1–10.

Love, P. E., Sing, C. P., Wang, X., Irani, Z., and Thwala, D. W. (2014). Overruns in transportation infrastructure projects. Structure and Infrastructure Engineering, 10(2):141–159.

Love, P. E., Sing, M. C., Ika, L. A., and Newton, S. (2019b). The cost performance of transportation projects: The fallacy of the Planning Fallacy account. Transportation Research Part A: Policy and Practice, 122(February):1–20.

Love, P. E., Zhou, J., Edwards, D. J., Irani, Z., and Sing, C. P. (2017). Off the rails: The cost performance of infrastructure rail projects. Transportation Research Part A: Policy and Practice, 99:14–29.

Love, P. E. D. and Ahiaga-Dagbui, D. D. (2018). Debunking fake news in a post-truth era: The plausible untruths of cost underestimation in transport infrastructure projects. Transportation Research Part A: Policy and Practice, 113:357–368.

Majd, S. and Pindyck, R. (1987). Time to build, option value, and investment decisions. Journal of Financial Economics, 18(1):7–27.

McDonald, R. and Siegel, D. (1986). The value of waiting to invest. The Quarterly Journal of Economics, 101(4):707–727.

Milne, A. and Whalley, A. E. (2000). 'Time to build, option value and investment decisions': a comment. Journal of Financial economics, 56(2):325–332.

Nguyen, L. D., Le-Hoai, L., Tran, D. Q., Dang, C. N., and Nguyen, C. V. (2019). Effect of project complexity on cost and schedule performance in transportation projects. Construction Management and Economics, 37(7):384–399.

Oriani, R. and Sobrero, M. (2008). Uncertainty and the market valuation of R&D within a real options logic. Strategic Management Journal, 29(4):343–361.

Pender, S. (2001). Managing incomplete knowledge: Why risk management is not sufficient. International Journal of Project Management, 19(2):79–87.

Pinheiro Catalão, F., Cruz, C. O., and Miranda Sarmento, J. (2019). Exogenous determinants of cost deviations and overruns in local infrastructure projects. Construction Management and Economics, 37(12):697–711.

Siddiqui, A. and Fleten, S. E. (2010). How to proceed with competing alternative energy technologies: A real options analysis. Energy Economics, 32(4):817–830.

Slade, M. E. (2001). Valuing managerial flexibility: An application of real-option theory to mining investments. Journal of Environmental Economics and Management, 41(2):193–233.

Terrill, M. (2016). Cost overruns in transport infrastructure — Grattan Institute.

Welde, M. and Odeck, J. (2017). Cost escalations in the front-end of projects–empirical evidence from Norwegian road projects. Transport Reviews, 37(5):612–630.

Zhang, J., Chen, F., and Yuan, X. X. (2020). Comparison of cost and schedule performance of large public projects under P3 and traditional delivery models: a Canadian study. Construction Management and Economics, 38(8):739–755.

Zidane, Y. J. and Andersen, B. (2018). The top 10 universal delay factors in construction projects. International Journal of Managing Projects in Business, 11(3):650–672.