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Restrictive clauses in loan contracts: A sequential Stackelberg game interpretation

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Abstract : We propose a dynamic stochastic game model to assess the value of restrictive clauses in private loan contracts. Restrictive clauses are used by lenders to mitigate their credit risk, for instance, by linking the interest payments of the borrower to some observable performance measure. We show how the value of these clauses for the lender and the borrower corresponds to the solution of a feedback Stackelberg game in discrete time. We consider two instances of restrictive clauses that are commonly found in the syndicated loan market, namely safety covenants and performance pricing, and provide numerical illustrations using representative instances to compare their relative efficiency.

Keywords : Stackelberg game, finance, loan contract, covenant, credit risk

Résumé : Cet article propose un modèle de jeu dynamique stochastique pour évaluer certaines clauses restrictives apparaissant dans des contrats de prêt. Les clauses restrictives sont utilisées par les prêteurs pour atténuer le risque de crédit, par exemple, en liant les paiements d'intérêts de l'emprunteur à une mesure de performance observable. L'article montre comment la valeur de ces clauses pour le prêteur et l'emprunteur correspond à la solution d'un jeu de Stackelberg en temps discret. Deux exemples de clauses restrictives couramment utilisées sur le marché des prêts syndiqués sont examinés : les covenants et la tarification selon la performance. Des illustrations numériques à partir d'instances représentatives sont fournies et sont utilisées pour comparer l'efficacité relative de telles clauses.

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1 Introduction

In the area of finance, there is a sizable literature using game-theoretical models to address a variety of corporate and investment issues (see, for instance, the surveys in [2] and [7]). One of the most frequently analyzed topics in this literature involves the strategic interactions between lenders and borrowers, mainly in the context of *credit risk*, that is, the risk that the borrower defaults on their contractual obligations. Issues addressed in this context include the possibility of strategic default, forced bankruptcy, successive contract renegotiation, asymmetric information, and various agency problems such as, for example, borrowed funds being invested in riskier ventures than originally intended.

In practice, restrictive clauses, in the form of *safety covenants* or *performance pricing*, are commonly found in the syndicated loan market, which represents roughly one-third of the overall international financing according to the Bank of International Settlement.

The literature on safety covenants suggests that restrictive clauses are the main tool to address agency problems between lenders and borrowers (see, for instance [1, 4, 11, 19, 29–31, 34, 36]. The literature on performance pricing covers the existence (see, for instance [3, 33]) and valuation [15, 24, 27, 33] of this type of restrictive clause.

The seminal model in [5] is the first contribution to the pricing of restrictive clauses in loan contracts. In this model, the loan contract includes a covenant that consists of maintaining a net positive equity value over the life of the loan. A covenant violation leads to forced bankruptcy, transferring the firm's ownership to the lender. Using some notable simplifications, the authors derive a closed-form solution, adapted from [26], and analyze the impact of the covenant on the value of the borrower's securities.

Most of the simplifications used in [5] are not consistent with the current practice on restrictive clause monitoring. Empirical findings [28, 29, 31, 32] show that most lenders do not exercise their right to force bankruptcy. Instead, they use a breach of a restrictive clause as bargaining power to renegotiate the loan contract's terms. On the other hand, borrowers are not passive actors in the process, as they have the possibility to reject new terms, or to modify their investment strategy.

In this paper, we interpret a loan contract including restrictive clauses as a dynamic stochastic game between the lender and the borrower, over a finite horizon corresponding to the contract's maturity. We consider two typical restrictive clauses: safety covenants (SC), which lead to successive "take it or leave it" renegotiations of the loan terms, and performance pricing (PP), which defines at inception how the terms will adjust to the evolution of some observable performance indicators.

In our model, the lender monitors the loan and can punish restrictive clause violations by altering the terms of the loan, or can adjust the interest rate according to some performance indicators; on the other hand, the borrower can choose to comply, exit, or alter the risk level of their operations. This strategic interaction can lead to successive adjustments to the conditions of the loan.

To the best of our knowledge, our model is the first to consider strategic default, successive contract renegotiation, and investment risk, in a dynamic setting. The strategic interactions between the lender and the borrower are modeled as a feedback Stackelberg game in discrete time. Our results show that safety covenants and performance pricing schedules do have a significant impact on the borrower's and on the lender's behavior, and consequently on the value of a loan.

The paper is organized as follows. Section 2 presents the general model, while Section 3 characterizes the equilibrium strategy and the value of the game. Section 4 proposes an illustrative implementation, specifying models for the state and debt processes, default probabilities and consequences, as well as the loan contract terms and restrictive clauses. Numerical results comparing the behavior and efficiency of SC and PP restrictive clauses are discussed in Section 5. Section 6 is a short conclusion.

2 Model and notation

We consider a loan contract between a *borrower* and a *lender*, with inception date $t = 0$ and maturity T. The loan contract includes restrictive clauses that need to be respected by the borrower, taking the form of constraints on one of the borrower's structural variables, assumed observable by both parties. The evolution of this variable is described by a stochastic process identified as the state process and denoted by S_t.

We assume different severity levels for the restrictive clauses, corresponding to *physical default* and to K *performance* modes. These severity levels are implemented via deterministic time-dependent barriers, typically determined from the loan outstanding value, and denoted respectively by b_t and $\hat{b}_{k_t}, k = 1, ..., K$, where $b_t < \hat{b}_{k_t} < \hat{b}_{k+1_t}$ for $k = 1, ...K - 1$ and $t \in [0, T]$.

According to the terms of the contract, the lender terminates the loan if the borrower's state variable hits the physical default barrier at any time. Accordingly, physical default is defined as a stopping time

$$\tau = \inf\{t \in (0, T] : S_t \leq b_t\}.$$

Moreover, the contract allows the lender to take some corrective actions at given monitoring dates (e.g. modify the physical default barrier and/or the interest rate) depending on the borrower's performance mode at that time. The borrower is considered in performance mode k at a given monitoring date t whenever $S_t \in (\hat{b}_{k-1_t}, \hat{b}_{k_t}], k = 1, ...K$, where $\hat{b}_{0_t} \equiv b_t$ and $\hat{b}_{K_t} = \infty$.

2.1 Time line

We evaluate loan contracts at inception $t_0 = 0$, taking the initial contract terms (spread, reimbursement schedule and restrictive clauses) as given parameters. Consequently, the outstanding debt at any date $t \in [0, T]$ is a deterministic process denoted by D_t.

At some fixed contractual dates $t_1, ..., t_{n-1}$, with $t_1 > 0$ and $t_{n-1} < t_n = T$, the lender will perform an audit to verify the level of the state variable with respect to the barriers $\hat{b}_{k_{t_m}}, k = 1, ..., K, m = 1, ..., n$. We denote by $\beta_m, m = 0, ..., n - 1$ the value, discounted at t_m, of one unit of money due at t_{m+1}. At a given monitoring date t_m, the lender, identified as the leader of the game (Player L) chooses among a set of available actions, and the borrower, identified as the follower (Player F), observing the actions implemented by the leader, reacts by choosing among a set of available responses.

The lender's set of actions

According to the performance mode, the lender can choose to implement a temporary interest rate modification, due at the next payment date and/or a temporary change of the physical default barrier (equivalent to a request for additional collateral). We denote by p the action vector of the lender. Note that the corrective measures implemented by the lender only apply until the next monitoring date, where the performance mode is re-evaluated.

The borrower's set of responses

At any monitoring date, the borrower can choose to refinance the loan, file for bankruptcy, or comply with the contract terms, eventually including the corrective actions implemented by the lender. In that latter case, the borrower can also decide to modify the risk level of their investment strategy. We denote by λ the action vector of the borrower.

The state dynamics

We assume that the state process $(S_t)_{0 \leq t \leq T}$ is Markovian and denote by $\mathbb{E}_m[\cdot]$ the expectation, conditional on no prior default and on the information available at t_m. This information includes the current level of the state process, along with the players' decision vector (p, λ). We also assume that both players' decisions may impact the evolution of the stochastic variable S_t. Accordingly, we denote by $0_m(s, p, \lambda)$ the probability of no physical default in the time interval (t_m, t_{m+1}), which depends on the current level s of the state variable at t_m, but also on the players' decision vector, observable at t_m.

2.2 Payoffs

At a given monitoring date t_m, $m = 1, ..., n-1$, the value $s = S_{t_m}$ of the state variable is observable by both players. At t_m, conditional on the borrower not being in physical default, each player will receive an immediate (expected) payoff that depends on the value of the state variable and on the decisions made by both players.

The immediate payoff of the lender (Player L) at t_m, $m = 1, ..., n-1$ is denoted by $w_m^L(s, p, \lambda)$. It includes the contractual interest payment and possibly the penalty imposed by the lender if the borrower complies, or the outstanding debt reimbursement if the borrower decides to refinance the loan. It also includes the lender's recovery upon default if the borrower decides to declare bankruptcy at t_m, or the expected value of their recovery in case physical default happens before the next monitoring date.

The immediate payoff of the borrower (Player F) at t_m, $m = 1, ..., n-1$ is denoted by $w_m^F(s, p, \lambda)$. It includes any additional revenue due to a change in investment strategy during the period $(t_m, t_{m+1}]$, the refinancing cost in case the borrower chooses to reimburse the loan, as well as the borrower's (expected) recovery upon default, either because they choose to declare bankruptcy at t_m, or in case physical default happens before the next monitoring date.

At maturity, the terminal payoff of the leader and of the borrower are denoted by $\vartheta^L(s)$ and $\vartheta^F(s)$.

2.3 Solution concept

From the strategic interactions between the lender and the borrower, the management of a loan contract including restrictive clauses gives rise to a game-theoretical interpretation, under a Stackelberg information structure, where the leader implements corrective actions at discrete monitoring dates as a function of the state variable, and where the follower's decisions depend on the state and on the leader's actions. Accordingly, the value of the loan for each player is obtained from the solution of a stochastic discrete-time Stackelberg game over a finite horizon, where both players use feedback strategies, and where the information available to the follower includes the decisions made by the leader.

3 Equilibrium strategies and value functions

A feedback strategy for a player $j \in \{L, F\}$ is a function δ_m^j indicating the decision made by Player j at monitoring date t_m, $m = 1, ...n-1$, as a function of the information available to this player. Accordingly,

$$\delta_m^L : s \to \mathcal{L}_m \tag{1}$$

$$\delta_m^F : (s, p) \to \mathcal{F}_m \tag{2}$$

where $\mathcal{L}_m$ (*resp.* $\mathcal{F}_m$) is the set of admissible decision vectors p for Player L (*resp.* admissible decision vectors λ for Player F), given the available information at t_m.

The functions used by the players to evaluate a strategy vector $\delta \equiv \left(\delta^L, \delta^F\right)$, where $j \in \{L, F\}$ and $\delta^j \equiv \left(\delta_m^j\right)_{m=1,...,n-1}$, satisfy the functional equations

$$V_m^j(s; \delta) = w_m^j(s, p, \lambda; \delta) + 0_m(s, p, \lambda; \delta)\, \beta_m \mathbb{E}_m\left[V_{m+1}^j\left(S_{t_{m+1}}; \delta\right)\right], m = 0, ...n-1 \tag{3}$$

$$V_n^j(s; \cdot) = \vartheta^j(s). \tag{4}$$

Given the dynamics of the state variable and (3)–(4), we seek a dynamic Stackelberg equilibrium in feedback strategies between the lender (leader) and the borrower (follower).

One of the issues in Stackelberg games is the indeterminacy that can arise when the follower has multiple optimal responses to a given action of the leader. In the context of a multi-stage feedback Stackelberg game, [8] characterizes the concepts of Strong Stackelberg and Weak Stackelberg equilibria (SSE and WSE), where a lexicographic order, based on the leader's outcome, is assumed when the follower has multiple best responses. The SSE case corresponds to a situation where the payoff of the leader is maximal, under the constraint that the follower's strategy is a best response, while the WSE gives rise to a security strategy for the leader, optimizing the worst case with regard to the follower's best response (see [9] for a discussion of the SSE).

In our model of the leader-follower interaction, we opt for the SSE assumption: when facing a tie, the follower will select an action that maximizes the outcome for the leader. This assumption is motivated by the fact that the two players' interests are not exactly opposed, both preferring strategies avoiding costly physical default. This assumption does not preclude the existence of multiple equilibria, but it ensures that the follower's response function is well-defined.

Accordingly, for a given function $v : \mathbb{R} \to \mathbb{R}$, define the reaction set of the follower to a decision vector p at $t_m, s = S_{t_m}$ for $m = 1, ..., n-1$ by

$$\mathcal{R}_m^F(s,p,v) \equiv \left\{ \lambda^* \in \arg\max_{\lambda\in\mathcal{F}} \left\{ w_m^F(s,p,\lambda) + 0_m(s,p,\lambda)\,\beta_m \mathbb{E}_m\left[v(S_{t_{m+1}}) \right] \right\} \right\}.$$

Definition 1. A strategy vector δ^* is a strong feedback Stackelberg equilibrium and $v_m^L(\cdot) \equiv V_m^L(\cdot;\delta^*)$ (resp. $v_m^F(\cdot) \equiv V_m^F(\cdot;\delta^*)$) is the corresponding equilibrium value function if the following conditions are satisfied for all s, p and for $m = 1, ..., n-1$:

$$\delta_m^{*F} \in \mathcal{R}_m^F(s,p,v_{m+1}^F) \tag{5}$$

$$v_m^L(s) = \max_{p\in\mathcal{L}_m} \left\{ \max_{\lambda\in\mathcal{R}_m^F(s,p,v_{m+1}^F)} \left\{ w_m^L(s,p,\lambda) + 0_m(s,p,\lambda)\beta_m \mathbb{E}_m\left[v_{m+1}^L(S_{t_{m+1}}) \right] \right\} \right\}. \tag{6}$$

As shown in [8], a feedback Stackelberg equilibrium in a multi-stage game is equivalent to a feedback Nash equilibrium in an associated game with twice the number of stages, where decisions are made by the leader in odd stages and by the follower in even stages (a switching-controller game), and where the state variable in even stages includes the actions selected by the leader, which are observable by the follower. A feedback Stackelberg equilibrium can then be characterized by the following dynamic program, which can be solved by backward induction from the known terminal value $\left(v_n^L, v_n^F\right)$.

$$v_n^L(s) = \vartheta^L(s) \tag{7}$$

$$v_n^F(s) = \vartheta^F(s) \tag{8}$$

For $m = 0, ..., n-1$

$$y_m^F(s,p) = \max_{\lambda\in\mathcal{F}_m} \left\{ w_m^F(s,p,\lambda) + 0_m(s,p,\lambda)\beta_m \mathbb{E}_m\left[v_{m+1}^F(S_{t_{m+1}}) \right] \right\} \tag{9}$$

$$\mathcal{R}_m^F(s,p) = \left\{ \lambda^* \in \arg\max_{\lambda\in\mathcal{F}_m} \left\{ w_m^F(s,p,\lambda) + 0_m(s,p,\lambda)\beta_m \mathbb{E}_m\left[v_{m+1}^F(S_{t_{m+1}}) \right] \right\} \right\} \tag{10}$$

$$y_m^L(s,p) = \max_{\lambda\in\mathcal{R}_m^F(s,p)} \left\{ w_m^L(s,p,\lambda) + 0_m(s,p,\lambda)\beta_m \mathbb{E}_m\left[v_{m+1}^L(S_{t_{m+1}}) \right] \right\} \tag{11}$$

$$v_m^L(s) = \max_{p\in\mathcal{L}_m} \left\{ y^L(s,p) \right\} \tag{12}$$

$$\mathcal{R}_m^L(s) = \left\{ p^* \in \arg\max_{p\in\mathcal{L}_m} \left\{ y^L(s,p) \right\} \right\} \tag{13}$$

$$v_m^F(s) = y^F(s,p^*), p^* \in \mathcal{R}_m^L(s). \tag{14}$$

The functions $v_m^j(s)$, $j \in \{L, F\}$ yield the equilibrium total expected discounted cash flows received by the lender (Player L) and the borrower (Player F) resulting from the loan contract, from date t_m until maturity, as a function of the level of the state variable at date t_m. Multiple equilibria may exist whenever the set $\mathcal{R}_m^L(s)$ is not a singleton at some m and s.

4 Implementation

In this section, we propose a specific implementation of our model, which will be used to illustrate the properties of an equilibrium solution and to analyze the two families of restrictive clauses that are the most frequently used in practice: safety covenants and performance pricing. To simplify the exposition, we assume in the sequel that players are risk-neutral and compute all expectations under the risk-neutral probability measure.

4.1 Loan contract

The loan contract between the lender and the borrower specifies the interest rate and the payment schedule over time, along with the restrictive clauses applying to some performance indicator. In our implementation, we assume that the restrictive clauses apply to the leverage ratio

$$L_t \equiv \frac{D_t}{S_t}$$

where S_t denotes the asset value of the follower. In that case, the physical default and performance mode barriers take the form

$$b_t = \alpha D_t \tag{15}$$

$$\hat{b}_{k_t} = \hat{\alpha}_k D_t. \tag{16}$$

In this particular implementation, we consider periodical interest payments (interest-only coupons). For simplicity, we assume that dates $t_m, m = 0, ..., n$, are equally spaced, with

$$\Delta \equiv t_{m+1} - t_m \text{ for } m = 0, ...n - 1,$$

and that coupon and monitoring dates coincide. We then have

$$\beta_m = \beta = exp(-r\Delta), m = 0, ..., n - 1 \tag{17}$$

$$D_{t_m} = D_0, m = 0, ..., n - 1 \tag{18}$$

$$c_m = D_0 i, m = 1, ..., n - 1 \tag{19}$$

$$c_n = D_0(1 + i) \tag{20}$$

where c_m is the contractual coupon payment due at t_m, i is the (periodic) contractual interest rate and r is the instantaneous risk-free rate, assumed constant.

The physical default barrier is then constant over time and proportional to the contractual debt (not including accrued coupons), so that

$$b_t = \alpha_\kappa D_0, \tag{21}$$

where $\kappa \in \{0, 1\}$ allows for a modification of the physical default barrier by the leader, with and $\alpha_1 > \alpha_0$. We set the contractual physical default parameter $\alpha_0 = 1$, which means that the physical default is triggered when the asset level attains the amount of the debt. This is also known as the *non-negative equity* condition [5].

4.2 Restrictive clauses

In addition to the right to force bankruptcy when the asset value hits the physical default barrier, we consider two types of restrictive clauses, audited at the contractual monitoring dates.

Safety covenant

A SC corresponds to a case where there are only two performance modes ($K = 2$), where mode 1 is called *technical default*. It is implemented by means of a technical default barrier (here constant)

$$\hat{b}_{t_m} = \hat{\alpha} D_{t_m} = \hat{\alpha} D_0, \tag{22}$$

where $\hat{\alpha} > \alpha_1$. Whenever the borrower's asset value is not above the technical default barrier at a monitoring date t_m, the lender can ask for an additional interest payment, due at the next monitoring date. We denote by $\hat{c}_m$ the coupon due at t_m, possibly modified from the contractual value c_m following a breach of the SC constraint.

The lender can also ask for more collateral, which is equivalent to raising the physical default barrier, by choosing $\kappa = 1$. When the borrower's asset value returns above the technical default barrier at a monitoring date t_m, both the interest due and the physical default barrier revert to their initial contractual value.[1]

Performance pricing

While an SC clause allows the lender to increase the interest payment whenever the covenant constraint is breached, PP can result in an increase or a decrease of the interest payment with respect to the initial coupon. PP is implemented by means of a collection of thresholds (the PP grid), and a function specifying the interest rate applicable according to the position of the performance indicator in the grid. Collateral increases are usually not included in PP clauses. The lender's strategy, that is, the PP grid and the corresponding interest-rate function, is reported as part of the contract.

In theory, an optimal PP schedule for the leader corresponds to the lender's equilbrium feedback strategy obtained from the solution of the dynamic program (7)–(14). In practice, PP uses monotone step functions that are independent of time and that define a constant interest rate on each interval of the PP grid. Accordingly, finding the optimal PP schedule requires approximating the lender's optimal strategy in the space of monotone step functions. The equilibrium value of this PP strategy for each player and the corresponding follower's response are then obtained by restricting the feasible actions of the leader to the contractual PP schedule in the dynamic program (7)–(14).

In this implementation, we assume that the PP grid is defined via a set of positive constants $\{\hat{\alpha}_k : k = 1, ..., K\}$ and corresponding rates $\{i_k : k = 1, ..., K\}$ so that the coupon $\hat{c}_{m+1}$ due at t_{m+1} is equal to $i_k D_0$ if, at the monitoring date t_m, $S_{t_m} \in (\hat{\alpha}_{k-1} D_0, \hat{\alpha}_k D_0], k = 1, ...K$, where $\hat{\alpha}_0 \equiv \alpha_0$ and $\hat{\alpha}_K = \infty$.

4.3 Asset process

The borrower is a firm with a capital structure that includes equity and debt. For simplicity, we assume that the debt is entirely financed by the lender. As long as the firm operates, it generates a stochastic pre-tax cash flow and pays a continuous dividend at a constant rate. In a risk-neutral context, the firm's asset value is the expected sum of the future cash flows, discounted at the risk-free rate. Following [26] and [21], we define a random variable X_t^l representing the evolution of the asset value between two monitoring dates, with dynamics

$$dX_t^l = (r - \varsigma) X_t^l dt + \sigma^l X_t^l dB_t, \tag{23}$$

where B_t is a standard Brownian motion, ς is the continuous dividend payout rate, and σ^l is the asset volatility in investment mode l. We consider two investment modes, where $l = 0$ corresponds to a "conventional" investment strategy, while $l = 1$ corresponds to an "aggressive" strategy, with

[1] Note that raising the physical default barrier above the borrower's current asset value is equivalent to forcing bankruptcy.

$\sigma_1 > \sigma_0$.[2] Assuming that physical default does not happen in (t_m, t_{m+1}), the value of the random variable $S_{t_{m+1}}$ at date t_{m+1} depends on the actions chosen by the lender and the borrower at t_m and satisfies

$$S_{t_{m+1}} = X^l_{t_{m+1}}|(S_{t_m}, \lambda) - (\hat{c}_{m+1})(1-\nu) \tag{24}$$

where the distribution of the random variable $X^l_{t_{m+1}}$, with l a component of λ and $X^l_{t_m} = S_{t_m}$, is obtained from Equation (23). The jump in the asset value accounts for the discrete coupon payments $\hat{c}_{m+1}$, adjusted for the net tax advantage of debt, where ν is the taxation rate.

Note that both the lender's and the borrower's decisions impact the evolution of the stochastic variable S_t. Any modification in the interest payments, which is a component of the decision variable p of the lender, directly affects the asset level, while the investment mode chosen by the borrower, which is a component of the decision variable λ, determines the volatility of the process between two monitoring dates.

4.4 Default model

According to the terms of the loan, a physical default event in (t_m, t_{m+1}) is a stopping time

$$\tau = \inf\{t \in (t_m, t_{m+1}) : S_t \leq \alpha_\kappa D_0\}.$$

The probability of no default event in a time interval (t_m, t_{m+1}),

$$\mathbb{P}(\tau \geq t_{m+1} | \tau > t_m),$$

given that S_t follows a Geometric Brownian Motion in the time interval (t_m, t_{m+1}) and that the barrier is the constant $\alpha_\kappa D_0$, is given by (see [5]):

$$\begin{aligned} 0_m(s,p,\lambda) \equiv& 1 - \mathbb{P}(\tau \in (t_m, t_{m+1})) \\ =& \Phi\left(\frac{-\frac{1}{\sigma_l}\log(\frac{\alpha_\kappa D_0}{s}) - \frac{\Delta}{\sigma_l}(r - \varsigma - \frac{\sigma_l^2}{2})}{\sqrt{\Delta}}\right) \\ &+ \left(\frac{\alpha_\kappa D_0}{s}\right)^{\left(\frac{2(r-\varsigma)}{\sigma_l^2}-1\right)} \Phi\left(\frac{\frac{1}{\sigma_l}\log(\frac{\alpha_\kappa}{s}) - \frac{\Delta}{\sigma_l}(r - \varsigma - \frac{\sigma_l^2}{2})}{\sqrt{\Delta}}\right), \end{aligned} \tag{25}$$

where Φ is the cumulative normal distribution, the indicator κ of the physical default barrier level is a component of p and the indicator l of the investment mode is a component of λ.

The amount recovered by the two players at date t when $S_t = s$, whether because of a physical default or when the follower decides to file for bankruptcy, is given by the recovery functions

$$R^L_t(s) \equiv \min\left\{(1-\gamma)\, s, \hat{D}_t\right\} \tag{26}$$

$$R^F_t(s) \equiv (1-\gamma)\, s - R^L_t(s), \tag{27}$$

where bankruptcy costs, characterized by the parameter γ, are assumed proportional to the asset level upon default. The outstanding debt at a date $t \in [t_m, t_{m+1})$, including the accrued coupon possibly adjusted for covenant violations, is given by

$$\hat{D}_t = D_0 + \hat{c}\frac{t - t_m}{\Delta} \text{ for } t \in [t_m, t_{m+1}), m = 0, ..., n-1. \tag{28}$$

[2]Using an aggressive strategy results in an additional payoff increasing the immediate reward of the borrower.

4.5 Parameter values and specific functions

Appendix A lists the base-case parameter values used in the numerical illustrations discussed in Section 5, including the PP grids. Some of these values are taken from the literature, while others are estimated from a sample of 5000 syndicated loans with restrictive clauses extracted from the Loan Pricing Corporation (LPC) DealScan database for the period 2000-2012.

For the terminal value, we consider the simplest scenario where the borrower reimburses all their debt at maturity. Under our assumption that debt is entirely financed by the lender, the value of equity is then equal to the asset value. In that case, if $S_T = s \geq 0$,

$$\vartheta^L(s) = \hat{c}_n \tag{29}$$

$$\vartheta^F(s) = s. \tag{30}$$

Otherwise, the terminal values for both players are obtained from Equations (26)–(27) with $D_T = \hat{c}_n$.

Several studies have explored the relationship between the leverage and the cost of refinancing. These works highlight how leverage is positively related to the perceived risk for lenders, thereby driving up borrowing costs [16, 17]. We use a piecewise linear specification, where the refinancing cost ω, applied to the amount D_0 to be refinanced at t_m, is linear in the debt-to-equity ratio above a given threshold L^*, and constant otherwise.This can be expressed as

$$\omega_m(s) = f_0 + \left(f_1 + f_2 \frac{D_0}{s - D_0}\right) \mathbb{I}\left(\frac{D_0}{s - D_0} > L^*\right), \tag{31}$$

where $s = S_{t_m}$ is the current asset value of the borrower. The threshold L^* and the constants f_0, f_1 and f_2 are obtained by calibrating this specification to our sample of syndicated loans. Their values are provided in Appendix A.

5 Illustrative examples

In this section, we report on numerical experiments performed using the specific implementation described in Section 4. These illustrative examples are chosen to depict how our stochastic dynamic game interpretation of loans with restrictive clauses can be used to reflect various empirically observed phenomena that are not captured by traditional models. Moreover, we use these experiments to compare how the nature of different restrictive clauses affects the behavior and the payoffs of the lender and the borrower.

Both PP and SC are commonly used in the syndicated loan market ([12] finds that around half of loan contracts in DealScan include PP clauses), and their respective efficiency in the context of credit risk has been abundantly investigated in the empirical finance literature (see, for instance, [13, 16, 17]). Safety covenants have the advantage of being more flexible, allowing the lender to renegotiate the contract terms over time, whenever the borrower is in technical default. Recent empirical studies highlight the increasing prevalence of SC and their strategic role in credit risk management [10, 14, 18].

On the other hand, PP has a wider application range, proposing a loan contract that adapts, positively or negatively, to changes in the credit worthiness of the borrower [15, 33]. Findings in [35] indicate that SC are more common among opaque borrowers, while PP tends to be used for rated companies. Moreover, [22] reports that the propensity to use PP increases by 10% when lenders have equity relationship with borrowers, while [35] finds that PP is predominant for long-maturity loans and, in general, when renegotiation costs are high.

In the following sections, we will address three issues that have been studied in the empirical literature: lenders' leniency, asset substitution, and credit-risk outcomes.

5.1 Lenders leniency

Many empirical studies find that most lenders do not exercise their right to force bankruptcy when the borrower breaches a security covenant (Roberts and Sufi, 2009; Nini et al., 2012; Roberts, 2015; Prilmeier, 2017). Our numerical experiments using the SC model support this observation, and show that the lender's decision to force bankruptcy or to punish breaches of covenants depends on the severity of the breach in a complex way.

The equilibrium strategy of the lender, as a function of the borrower's asset value, is represented in Figure 1 for various auditing dates. Recall that the lender can only intervene when the borrower is in technical default (here, for asset values between 100 and 130). Figure 1 plots the temporary increase in the interest rate as a function of the asset value. Solid lines are used in the regions where the physical default barrier is equal to 100, while dotted lines are used in the regions where the lender chooses to increase the physical default barrier to 110. The regions where no punishment is plotted correspond to forced bankruptcy.[3]

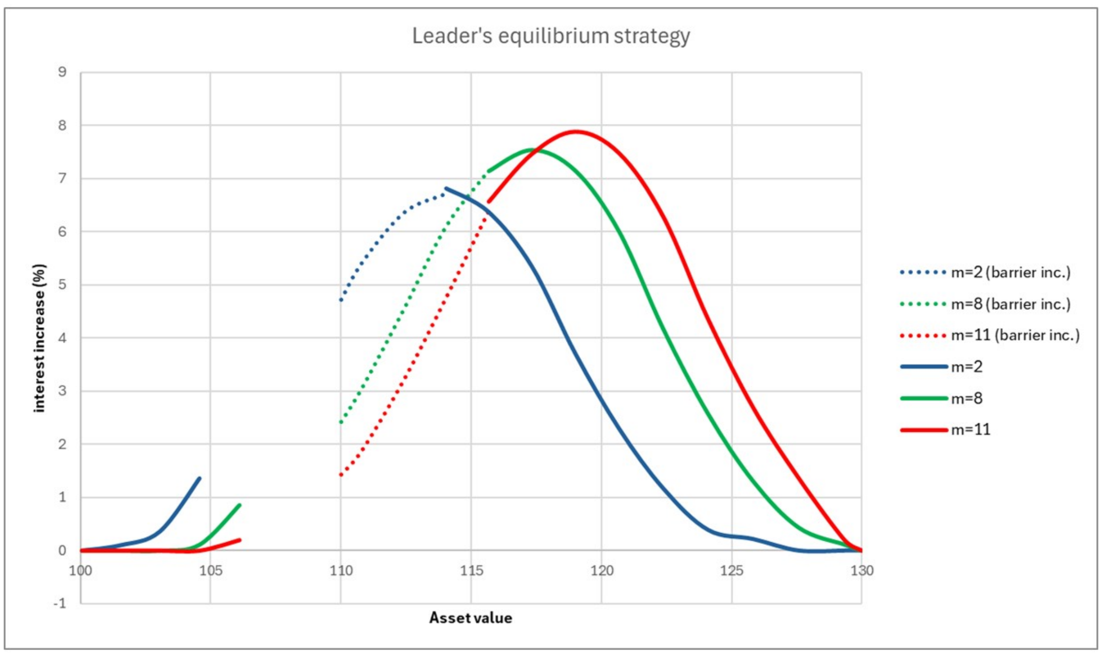

Figure 1: Leader's equilibrium strategy as a function of the asset value, at some selected auditing dates, under the base-case specification given in Table A1. Solid lines indicate the additional interest (punishment) in the regions where the default barrier is equal to 100. Dotted lines indicate the interest punishment in regions where the default barrier is increased by the leader. The leader forces bankruptcy in the regions where no punishment is reported.

Figure 1 shows that the leader is optimally tolerating some levels of technical default without any form of punishment. This happens for asset values close to the initial physical default barrier, and the tolerance region becomes wider as time to maturity decreases. This lenient behavior of the leader in equilibrium can be conceptually related to the strategic debt servicing theory, where successive renegotiations may lead the lender to reduce the coupon, particularly when bankruptcy costs are important [25].

Out of the leniency region, the interest punishment is not monotone: it increases up to a maximum value that decreases with time to maturity, then decreases, taking the shape of a bell. At some asset value inside the increasing punishment region, the leader forces bankruptcy. For higher asset values

[3]The lender can impose immediate bankruptcy by raising the physical default barrier above the borrower's current asset value.

still inside the increasing punishment region, the leader increases the physical default barrier along with the interest punishment. Note that both punishment instruments are likely to increase the default probability and decrease the loss given default. At some intermediate value between the physical and technical default barriers, the leader no longer increases the default barrier, and then the interest punishments start to decrease, vanishing as the asset value reaches the technical default barrier. In that region, the default probability is smaller; not only is the lender not using their option to force bankruptcy, they are reducing the punishment of a breach of covenant, thus reducing the default probability and increasingly allowing the borrower's asset value to return above the technical default barrier.

It is interesting to note that the equilibrium strategy of the leader under a safety covenant differs significantly from the interest adjustments typically implemented in leverage-based schedules, such as performance pricing, where the interest rate is expected to increase with leverage (see for instance the PP schedule illustrated in Figure 2).

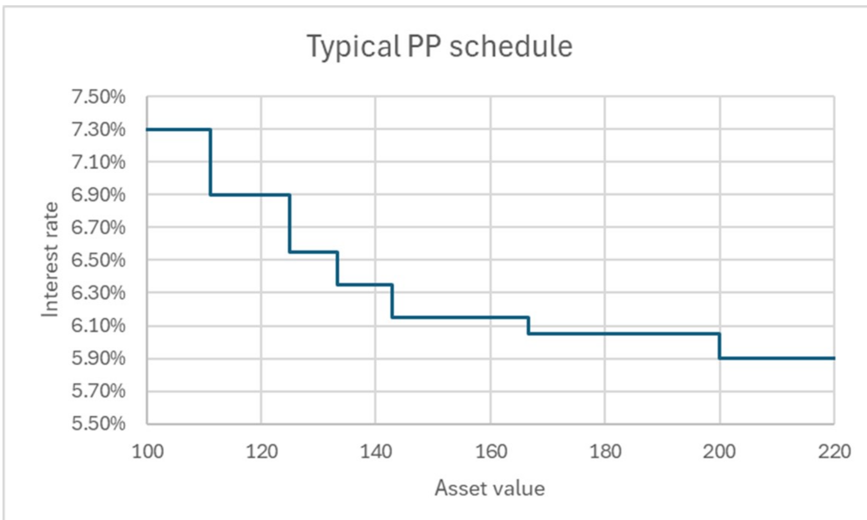

Figure 2: Typical PP interest rate as a function of the borrower's asset value, under the base-case specification given in Table A1.

5.2 Asset substitution

Asset substitution is one of the most important agency problems in finance. By shifting from low-risk to high-risk projects with higher earning opportunities, a borrower can increase their expected revenue at the expense of an increase in their credit risk, thereby negatively impacting the loan value. Safety covenants have been identified as the main instrument to address agency problems between lenders and borrowers in the early literature [1, 30, 31, 34], while later works (for instance [20]) identify PP as a particularly efficient way to reduce the asset substitution problem.

In our setting, asset substitution is captured by the possibility for the borrower to opt for an "aggressive" investment strategy, yielding an immediate additional earning, expressed as a proportion g of the current asset value, as well as a modification in the volatility of the asset value process ($\sigma_1 > \sigma_0$).

We use our model of strategic interaction to obtain illustrative results of the use of asset substitution in equilibrium under four scenarios: no restrictive clause, security covenant (SC), and performance pricing (PP1 and PP2). For performance pricing, we design two distinct grids; scenario PP1 corresponds to a grid with equally-spaced performance modes that yields the same value as SC to the leader, while scenario PP2 corresponds to a grid with the same number of performance modes obtained from the Deal Scan database. Both PP grids are provided in Appendix A.

Results corresponding to the four scenarios are presented in Figures 3 to 6. These figures depict the borrower's equilibrium strategy as a function of the asset value at some audit dates, that is, the best response of the borrower to the leader's equilibrium strategy. In all scenarios, we assume that bankruptcy is triggered by the asset level attaining the amount of the debt. In the SC scenario, bankruptcy can also happen at higher values of the asset level, namely in the region where the physical default barrier is modified by the lender.

Examination of Figure 3 shows that the borrower will generally adopt asset substitution, except when their asset value is very low, where they will play conservatively to reduce the probability of default due to the larger asset volatility corresponding to an aggressive investment strategy. The asset substitution region increases as maturity approaches.

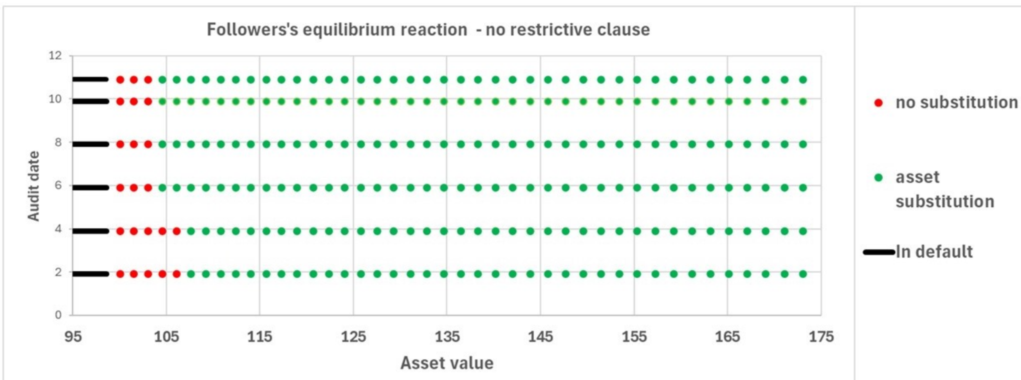

Figure 3: Follower's strategy in equilibrium as a function of their asset value (horizontal axis) for various audit dates (vertical axis) when the contract does not include restrictive clauses. Parameter values are given in Table A1.

In contrast, under scenario SC (Figure 4), the borrower invests aggressively in distinct regions: when their asset value is far above the technical default barrier and the risk of technical default is small; when their asset value is just below the technical default barrier, thus trying to escape the punishment region; and when their asset value is just below the threshold where the lender would increase the physical default barrier, thereby forcing bankruptcy. This seemingly counterintuitive behavior can be explained by the fact that, in that region, bankruptcy is almost certain, while the immediate additional dividend from asset substitution belongs to the borrower and will be protected in case of default.

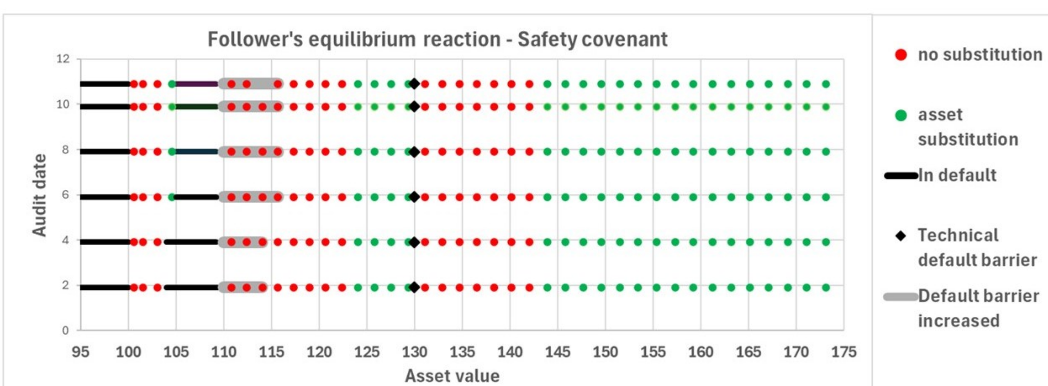

Figure 4: Follower's strategy in equilibrium as a function of their asset value (horizontal axis) for various audit dates (vertical axis) under a security covenant. Parameter values are given in Table A1.

Again, the asset substitution regions become wider as maturity approaches. It is interesting to note that the borrower is systematically adopting asset substitution as the asset value approaches the technical default barrier from below, showing that the equilibrium strategy admits asset substitution in the technical default region. On the other hand, Figure 4 shows that the borrower does invest conservatively when the asset value approaches the technical barrier from above. Comparing Figures 3 and 4 shows a reduction in the area where asset substitution takes place when the contract includes a safety covenant.

We now investigate the borrowers equilibrium strategy under performance pricing. Figures 5 and 6 corresponding to different PP grids show qualitatively similar results, and these results are also qualitatively similar to the ones obtained under scenario SC around the technical default barrier: asset substitution is used by the borrower when their asset value approaches a threshold separating two performance modes from below, and a conservative strategy is used when their asset value approaches a threshold from above. At some sufficiently high asset value that depends on the PP grid configuration,[4] performance pricing ceases to deter asset substitution. As in the SC scenario, the size of the asset substitution regions increases as maturity approaches. There is no significative difference between the size of the asset substitution regions among scenarios SC, PP1 and PP2.[5]

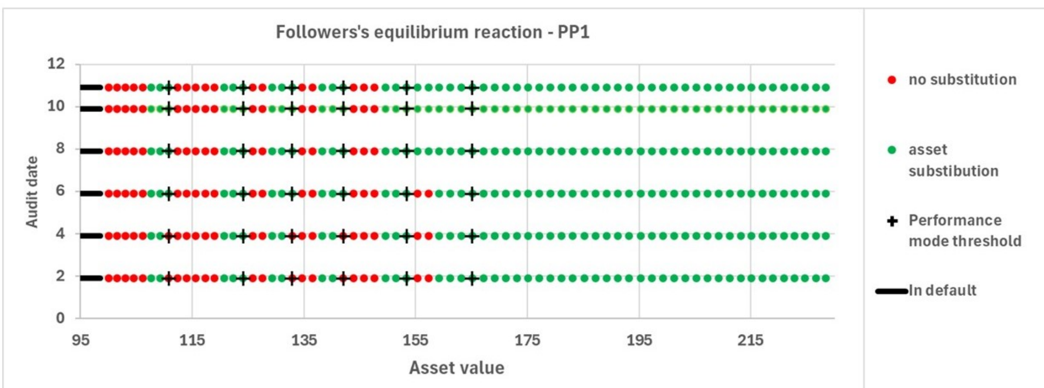

Figure 5: Follower's strategy in equilibrium as a function of their asset value (horizontal axis) for various audit dates (vertical axis) under the performance pricing grid given in Table A2. Other parameter values are given in Table A1.

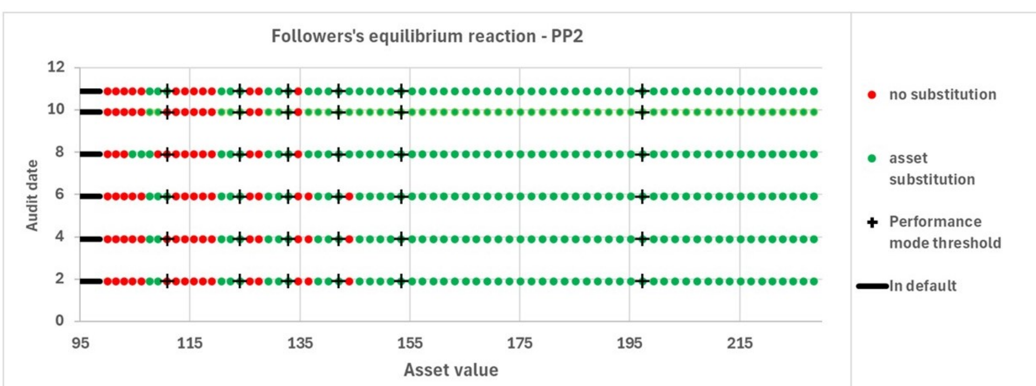

Figure 6: Follower's strategy in equilibrium as a function of their asset value (horizontal axis) for various audit dates (vertical axis) under the performance pricing grid given in Table A2. Other parameter values are given in Table A1.

[4]In scenario PP2, the borrower tends to use asset substitution when variations in spreads from one mode to the other are lower.

[5]However this is not a reliable metric for the efficiency of these scenarios in preventing asset substitution since it does not account for the probability of the asset value reaching these areas.

The results of our investigation about the efficiency of restrictive clauses is robust to changes in parameter values. Clearly, the impact of restrictive clauses depends on the profitability g of asset substitution and on its relative level of risk σ_1/σ_0; we find that the effectiveness of restrictive clauses in preventing asset substitution decreases with its profitability and increases with the associated risk. On the other hand, opportunities for asset substitution may change over the course of the contract, making it practically impossible to eliminate asset substitution using only financial covenants, as supported by the empirical literature.

5.3 Credit-risk outcomes

Restrictive clauses are fundamentally meant to mitigate credit risk. In that respect, [24] reports that, for leveraged borrowers, PP can lead to a higher probability of bankruptcy than SC and argues that this is due to the lack of flexibility of PP schedules with respect to contract renegotiation. In this section, we investigate the relative efficiency of restrictive clauses with respect to credit risk by comparing not only the default probabilities, but also the loss given default (LGD) and the unconditional expected loss.

These values are obtained by Monte Carlo simulation, using the asset price model and the equilibrium strategies computed by the dynamic program (7)–(14), where the lender's strategy is adjusted to account for four different scenarios: no restrictive clause (the lender has no option), security covenant (the lender can choose to increase the interest rate or the physical default barrier whenever the borrower's asset value is lower than the technical default barrier), interest-only covenant (the lender cannot raise the physical default barrier), and performance pricing (the lender's actions are selected according to the the PP1 schedule).

Note that the contractual spread $(i - r)$ depends on the leverage at inception, or, equivalently for a fixed debt level, on the initial asset value. Figures 7 and 8 report the credit-risk outcomes corresponding to various initial asset values, where contractual spreads are derived from the closed-form solutions in [26] and [5], thus accounting, at least approximately, for the risk-mitigating impact of restrictive clauses.

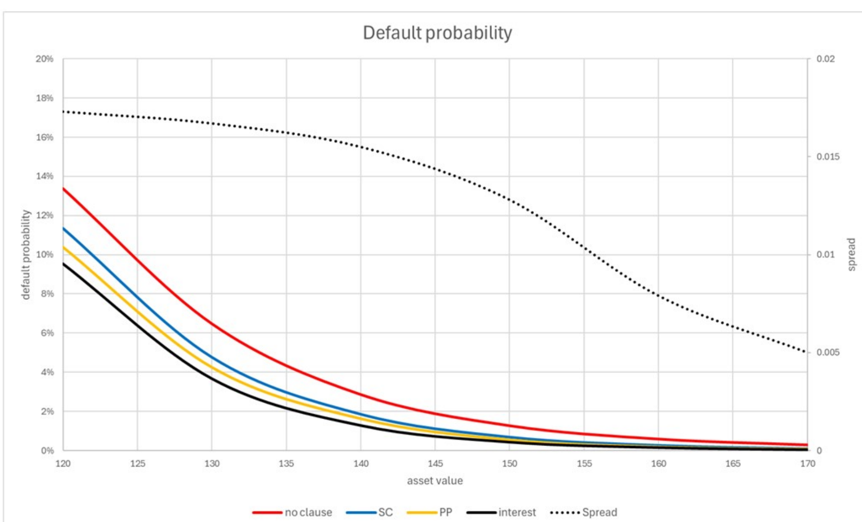

Figure 7: Default probability as a function of the initial asset value for various restrictive clauses (left axis). The contractual spread corresponding to the initial asset value is represented on the right axis. Other parameter values are given in Tables A1 and A2.

Figure 7 confirms the risk-mitigating impact of financial covenants, the highest default probability occurring when the debt contract does not contain any restrictive clause. With respect to the default probability, performance pricing performs better than the security covenant, but is dominated by the interest-only covenant. This result confirms many empirical observations based on default or bankruptcy occurrences. However, it may seem counter-intuitive at first sight, given that the PP grid is sub-optimal and that the interest-only covenant obtains by removing one of the leader's strategic instruments.

In reality, the players' objective is not to minimize the probability of default, but rather to maximize their expected gain, which should be better aligned with their expected loss from default outcomes. Given that the debt level is constant in our implementation, the LGD is the same for all scenarios where the physical default threshold is constant, and corresponds to the bankruptcy costs. In scenario SC, the possibility for the leader to raise the physical default barrier allows to significantly reduce the LGD in equilibrium (Figure 8, left panel). As a consequence, the unconditional expected loss is lower in the SC scenario, followed by the interest-only covenant, PP, and, finally, the loan contract without restrictive clauses (Figure 8, right panel). These results show that restrictive clauses do have an impact on credit-risk outcome, reducing significantly both the probability of default and the expected loss from default.

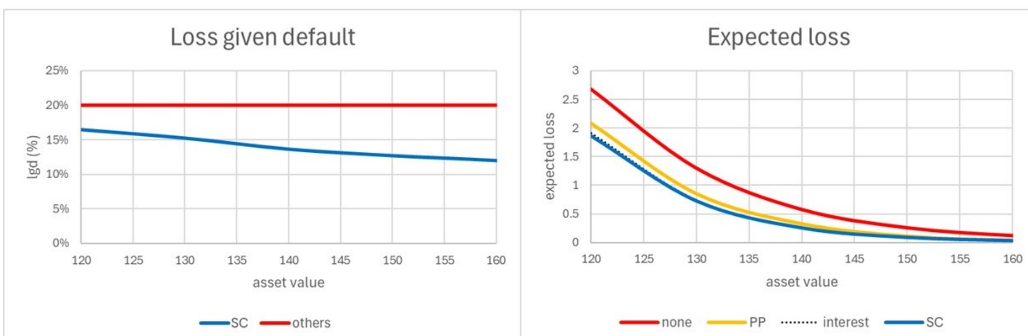

Figure 8: Loss given default and expected loss as a function of the initial asset value for various restrictive clauses and risk-adjusted spreads. Other parameter values are reported in Tables A1 and A2.

Clearly, one of the crucial determinant of credit-risk outcomes is the level of the physical default barrier, characterized in our model by the parameter α_0, which can also be increased through monitoring when provided for by a security covenant. In the empirical literature, there is no consensus on what should be the appropriate default barrier. For instance, [5] sets it to the present value of the outstanding loan (assuming that the debt is reimbursed in a single payment at maturity), [6] and [23] use the outstanding loan value at the default time, while [21] assumes an endogenous default barrier optimizing the equity value.

Although a higher default barrier increases the probability of default, it also protects the lender by reducing the LGD and thus providing a higher minimal value to the loan. On the other hand, the LGD is directly linked to the bankruptcy costs, which therefore impact the credit-risk outcomes [5].

We illustrate this complex relationship in Figure 9, where we use our implementation of the game to compare the equilibrium value of a loan under a security covenant, as a function of α_0, for various bankruptcy cost levels. Results are obtained under the base case described in Table A1.[6]

For any bankruptcy cost level, the borrower's equilibrium value (right panel) is relatively stable when $\alpha_0 < 1$, and then decreases as the default probability increases with the contractual default barrier. On the other hand, the lender's equilibrium value behavior changes with the level of bankruptcy

[6]Results corresponding to the performance pricing clause are qualitatively very similar.

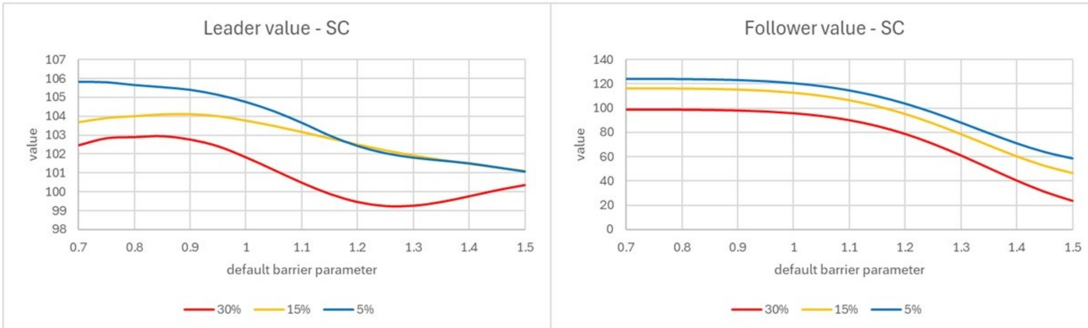

Figure 9: Leader and follower equilibrium value of a loan contract including a security covenant. Values are reported at inception as a function of the default barrier parameter α_0 for various bankruptcy cost levels, expressed as a percentage of the asset value at the time of default. Other parameter values are given in Table A1.

cost: For low bankruptcy costs, the leader should choose a barrier much lower than the outstanding loan value, whereas for high bankruptcy costs, the leader should either choose a barrier much higher than the outstanding debt value to cover the bankruptcy cost, or a low default barrier to lower the default probability.

Overall, selecting the contractual default barrier remains a challenging exercise. The possibility for the leader to change this barrier when the borrower is in technical default may be interesting in terms of risk management, however our investigation shows that this option does not improve the loan value in all situations.

6 Conclusion

This paper provides a game-theoretic model of the interactions between the lender and the borrower when the debt contract includes restrictive clauses, such as security covenants or performance-pricing clauses. Such restrictive clauses are commonly found in contracts involving sizable amounts, namely in the syndicated loan market, in order to mitigate credit risk.

Given that restrictive clauses provide the lender with instruments to adjust the terms or to terminate the contract, namely by modifying the interest rate and/or the bankruptcy conditions, we model the interactions between the lender and the borrower as a dynamic stochastic Stackelberg game where players use feedback strategies. Our model accounts for the various possibilities available to the borrower as a response to the actions taken by the lender, including asset substitution, which is one of the most important agency problems in the credit-risk literature. Using a representative implementation, we show how a game-theoretic model can be used to compute the value of a loan and to characterize the equilibrium strategies of the lender and the borrower, as a function of time and of the observed value of a performance indicator. Results from a numerical experiments are used to illustrate and compare salient features of two commonly-used restrictive clauses. We show that our model is able to explain various observations from the empirical finance literature.

Our paper provides a realistic implementation of dynamic games in the area of finance. The feedback-Stackelberg model developed in this paper can be used, not only to price restrictive clauses, but also to design efficient contracts beneficial to both parties and to the smooth functioning of the loan market.

A Appendix

Table A1 lists the base-case parameter values used in the numerical illustrations. Table A2 defines the performance schedule corresponding to Scenarios PP1 and PP2.

Table A1: Base-case model parameter values

Parameters	Notation	Base-case value
Amount of the loan	D_0	100
Leverage at inception	$\frac{D_0}{S_0}$	$\frac{2}{3}$
Loan maturity (years)	T	1
Number of monitoring dates	n	12
Risk-free rate	r	3%
Contractual (monthly) interest rate	i	4.5 %
Net tax advantage of debt	ν	35%
Bankruptcy costs	γ	20%
Dividend payout rate	ς	1%
Asset volatility (annual) according to l	σ_0, σ_1	15%, 25%
Immediate revenue when l=1 (% of the asset value)	g	$\frac{1}{12}$ %
Physical default parameters	α_0, α_1	1, 1.1α_0
Technical default parameter	$\hat{\alpha}$	1.3 α_0
Refinancing cost parameters	f_0, f_1, f_2	0.005446, -0.01699, 0.0289
Refinancing cost threshold	L^*	0.6

Table A2: Performance modes and corresponding interest rates

Scenario					**PP grid**				
	k	0	1	2	3	4	5	6	K
PP1	α_k	1	1.111	1.25	1.333	1.429	1.538	1.667	∞
	$i_k - r$	–	5.2%	4.7%	4.1%	3.5%	3%	2.6%	2.35%
PP2	α_k	1	1.111	1.25	1.333	1.429	1.667	2.00	∞
	$i_k - r$	–	4.30 %	3.90 %	3.55 %	3.35 %	3.15 %	3.05 %	2.90 %

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